# Estimating variance

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# Now we can make predictions

Now we are dangerous.

# Predictions are useless without uncertainty

- We are doing statistics
- We want to know about **uncertainty**
- This is the most useful part of the analysis

# What do we want the uncertainty for?

- Variance of total abundance
- Map of uncertainty (coefficient of variation)

# Where does uncertainty come from?

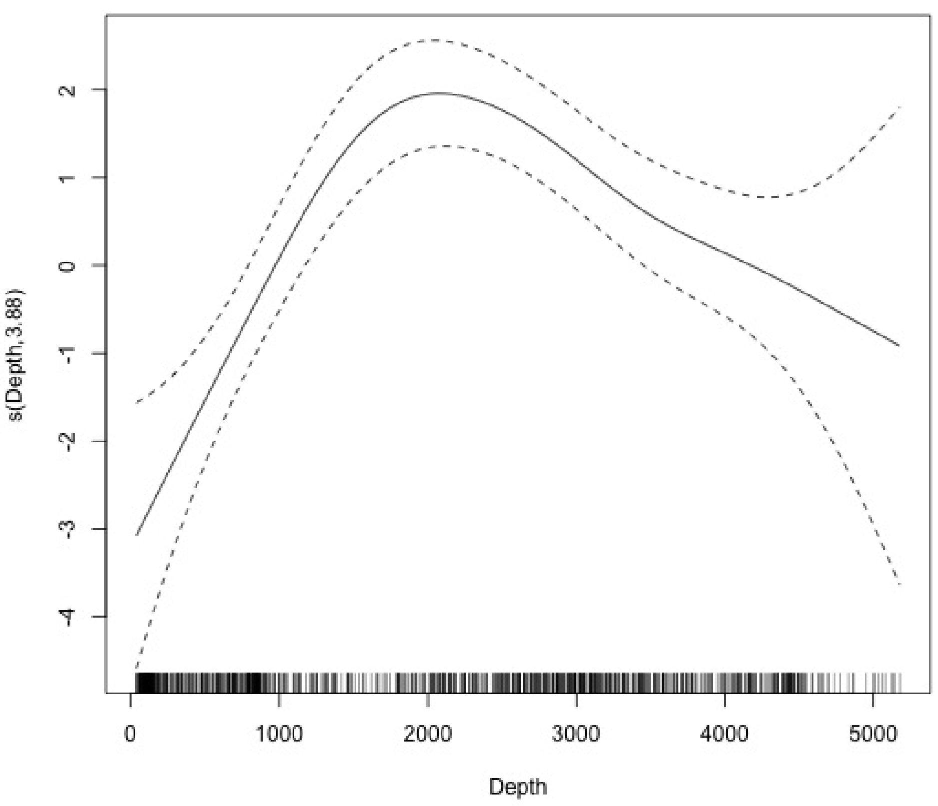
# Sources of uncertainty

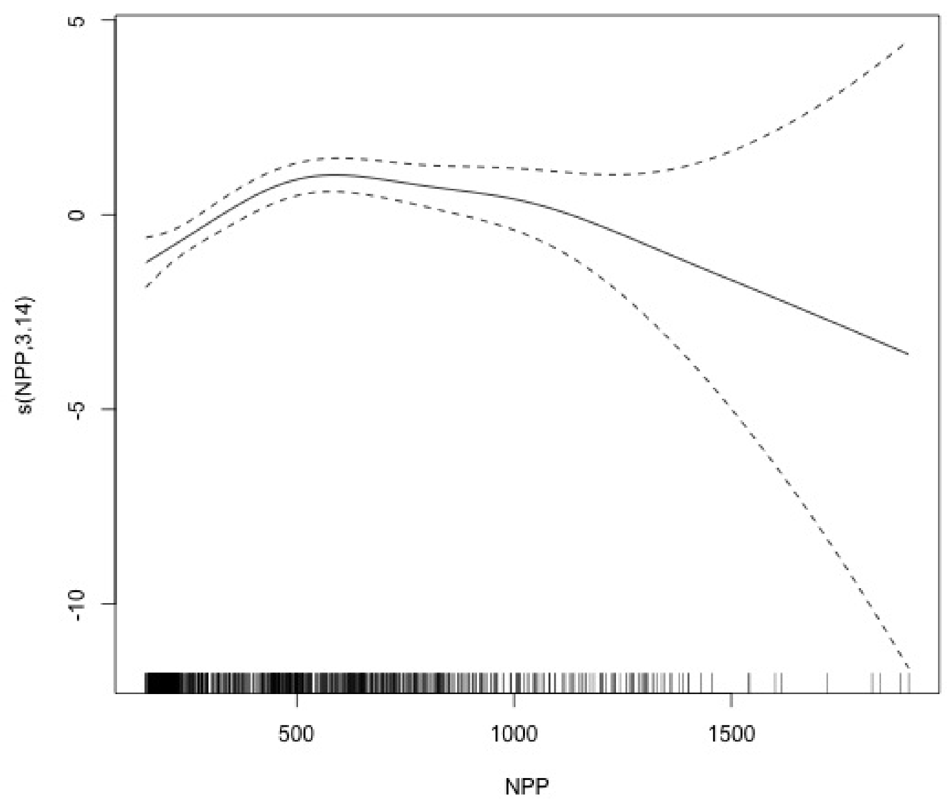
- Detection function
- GAM parameters

# Let’s think about smooths first

# Uncertainty in smooths

- Dashed lines are +/- 2 standard errors
- How do we translate to $\hat{N}$?



# Back to bases

- Before we expressed smooths as:
  - $s(x) = \sum_{k=1}^{K} \beta_k b_k(x)$
- Theory tells us that:
  - $\boldsymbol{\beta} \sim N(\hat{\boldsymbol{\beta}}, \mathbf{V}_{\boldsymbol{\beta}})$
  - where $\mathbf{V}_{\boldsymbol{\beta}}$ is a bit complicated
- Apply parameter variance to $\hat{N}$

# Predictions to prediction variance (roughly)

- "map" data onto fitted values $\mathbf{X}\boldsymbol{\beta}$
- "map" prediction matrix to predictions $\mathbf{X}_{\mathrm{p}}\boldsymbol{\beta}$
- Here $\mathbf{X}_{\mathrm{p}}$ need to take smooths into account
- pre-/post-multiply by $\mathbf{X}_{\mathrm{p}}$ to "transform variance"
  - $\Rightarrow \mathbf{X}_{\mathrm{p}}^{\mathrm{T}}\mathbf{V}_{\boldsymbol{\beta}}\mathbf{X}_{\mathrm{p}}$
  - link scale, need to do another transform for response

# Adding in detection functions

# GAM + detection function uncertainty

(Getting a little fast-and-loose with the mathematics)

From previous lectures we know:

$$\text{CV}^2\left(\hat{\text{N}}\right) \approx \text{CV}^2(\text{GAM}) +$$

$$\text{CV}^2\left(\text{detection function}\right)$$

# Not that simple...

- Assumes detection function and GAM are **independent**
- Maybe this is okay?

# A better way (for some models)

- Include the detectability as a “fixed” term in GAM
- Mean effect is zero
- Variance effect included
- Uncertainty “propagated” through the model
- Details in bibliography (too much to detail here)

# That seemed complicated…

# R to the rescue

# In R...

- Functions in `dsm` to do this
- `dsm.var.gam`
  - assumes spatial model and detection function are independent
- `dsm.var.prop`
  - propagates uncertainty from detection function to spatial model
  - only works for `count` models (more or less)

# Variance of abundance

## Using `dsm.var.gam`

```
dsm_tw_var_ind <- dsm.var.gam(dsm_all_tw_rm, predgrid,
off.set=predgrid$off.set)
summary(dsm_tw_var_ind)
```

```
Summary of uncertainty in a density surface model calculated
 analytically for GAM, with delta method

Approximate asymptotic confidence interval:
      5%     Mean      95%
1538.968 2491.864 4034.773
(Using delta method)

Point estimate                 : 2491.864
Standard error                 : 331.1575
Coefficient of variation       : 0.2496
```

# Variance of abundance

## Using dsm.var.prop

```
dsm_tw_var <- dsm.var.prop(dsm_all_tw_rm, predgrid,
off.set=predgrid$off.set)
summary(dsm_tw_var)
```

```
Summary of uncertainty in a density surface model calculated
 by variance propagation.

Quantiles of differences between fitted model and variance model
      Min.    1st Qu.     Median       Mean    3rd Qu.       Max.
-4.665e-04 -3.535e-05 -4.358e-06 -3.991e-06  2.095e-06  1.232e-03

Approximate asymptotic confidence interval:
      5%      Mean      95%
1460.721 2491.914 4251.075
(Using delta method)

Point estimate                 : 2491.914
Standard error                 : 691.8776
Coefficient of variation       : 0.2776
```

# Plotting - data processing

- Calculate uncertainty per-cell
- `dsm.var.*` thinks `predgrid` is one "region"
- Need to split data into cells (using `split()`)
- (Could be arbitrary sets of cells, see exercises)
- Need `width` and `height` of cells for plotting

# Plotting (code)

```
predgrid$width <- predgrid$height <- 10*1000
predgrid_split <- split(predgrid, 1:nrow(predgrid))
head(predgrid_split,3)
```

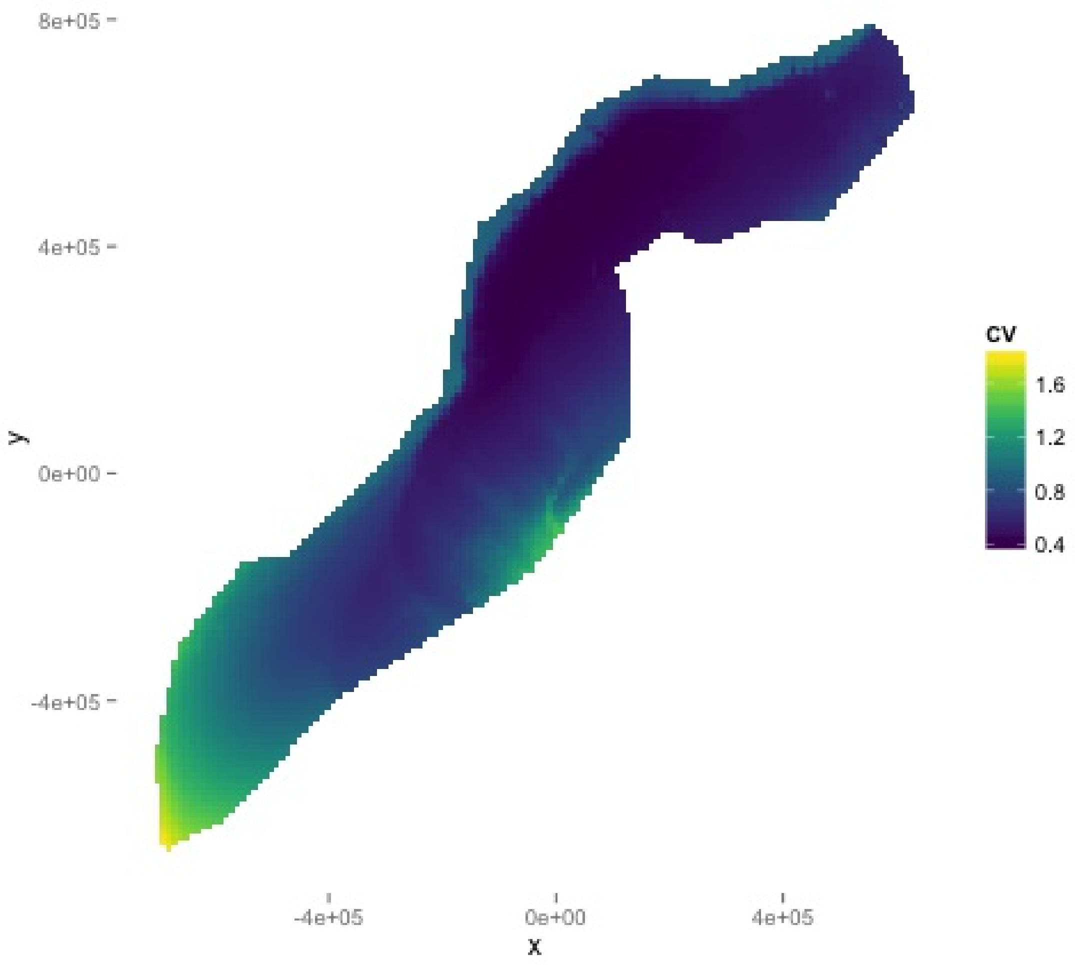
```
$`1`
           x      y    Depth      SST      NPP off.set height width
126 547984.6 788254 153.5983 9.04917 1462.521   1e+08  10000 10000

$`2`
           x      y    Depth       SST     NPP off.set height width
127 557984.6 788254 552.3107 9.413981 1465.41   1e+08  10000 10000

$`3`
           x      y    Depth       SST      NPP off.set height
width
258 527984.6 778254 96.81992 9.699239 1429.432   1e+08  10000
10000
```

```
dsm_tw_var_map <- dsm.var.prop(dsm_all_tw_rm, predgrid_split,
                               off.set=predgrid$off.set)
```

# CV plot

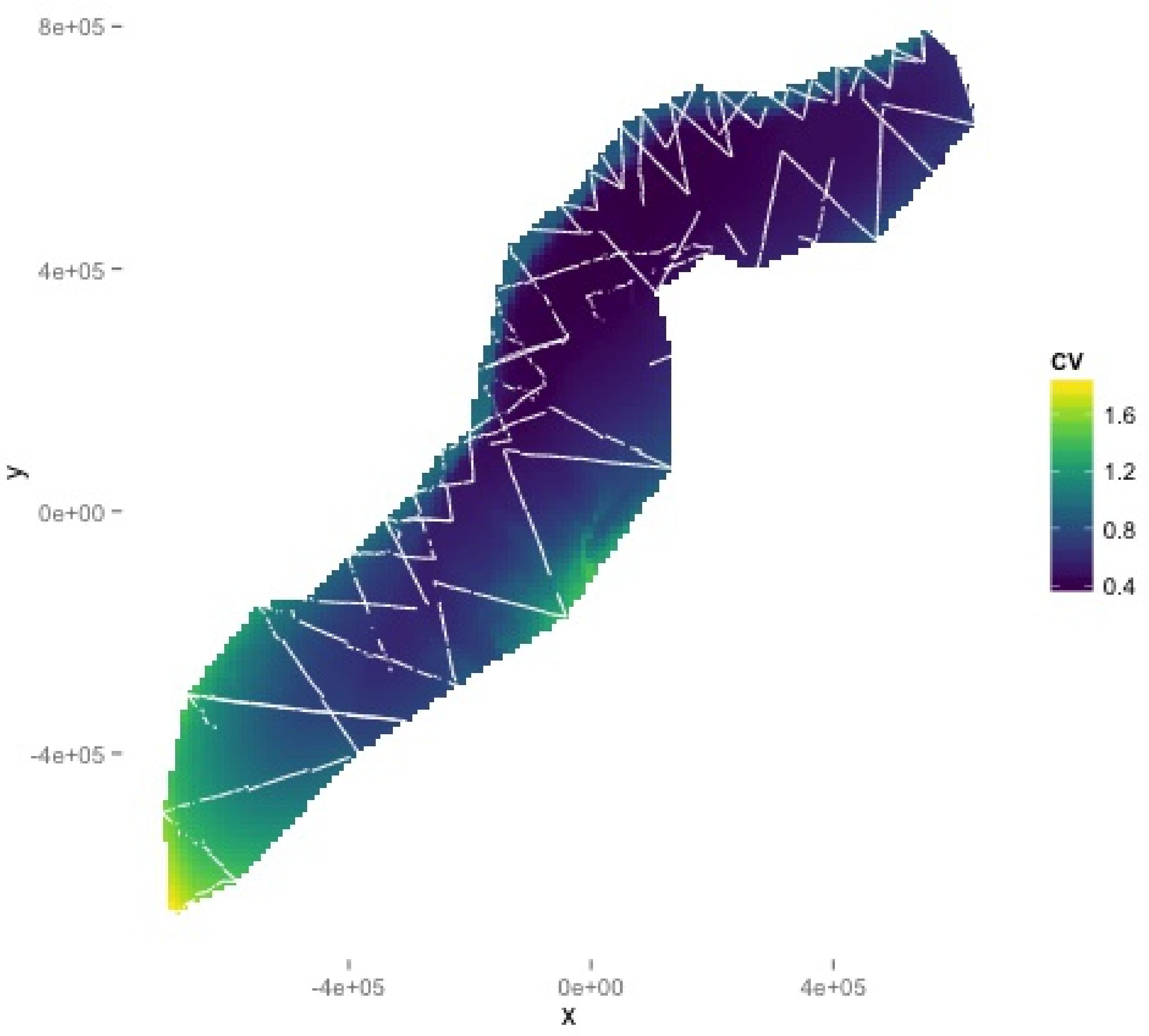



```
p <- plot(dsm_tw_var_map,
observations=FALSE, plot=FALSE)
+
        coord_equal() +
        scale_fill_viridis()
print(p)
```

# Interpreting CV plots

- Plotting coefficient of variation
- Standardise standard deviation by mean
- $\text{CV} = \text{se}(\hat{\text{N}})/\hat{\text{N}}$ (per cell)
- Can be useful to overplot survey effort

# Effort overplotted



# Big CVs

- Here CVs are "well behaved"
- Not always the case (huge CVs possible)
- These can be a pain to plot
- Use `cut()` in R to make categorical variable
  - e.g. `c(seq(0,1, len=100), 2:4, Inf)` or somesuch

# Recap

- How does uncertainty arise in a DSM?
- Estimate variance of abundance estimate
- Map coefficient of variation

# Let’s try that!