Lecture 2 : Generalized Additive Models

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Overview

- The count model, from scratch
- What is a GAM?
- What is smoothing?
- Fitting GAMs using `dsm`

Building a model, from scratch

- Know count n_j in segment j
- Want :

$$n_j = f([\text{environmental covariates}]_j)$$

- Additive model of smooths s:

$$n_j = \exp\left[\beta_0 + s(\text{y}_j) + s(\text{Depth}_j)\right]$$

- model terms
- exp is the *link function*

Building a model, from scratch

- What about area and detectability?

$$n_j = A_j \hat{p}_j \exp\left[\beta_0 + s(\mathrm{y}_j) + s(\mathrm{Depth}_j)\right]$$

- A_j area of segment - "offset"
- $\hat{p}_j$ probability of detection in segment

Building a model, from scratch

- It's a statistical model so:

$$n_j = A_j \hat{p}_j \exp\left[\beta_0 + s(\text{y}_j) + s(\text{Depth}_j)\right] + \epsilon_j$$

- n_j has a distribution (count)
- ϵ_j are *residuals* (differences between model and observations)

That's a Generalized Additive Model!

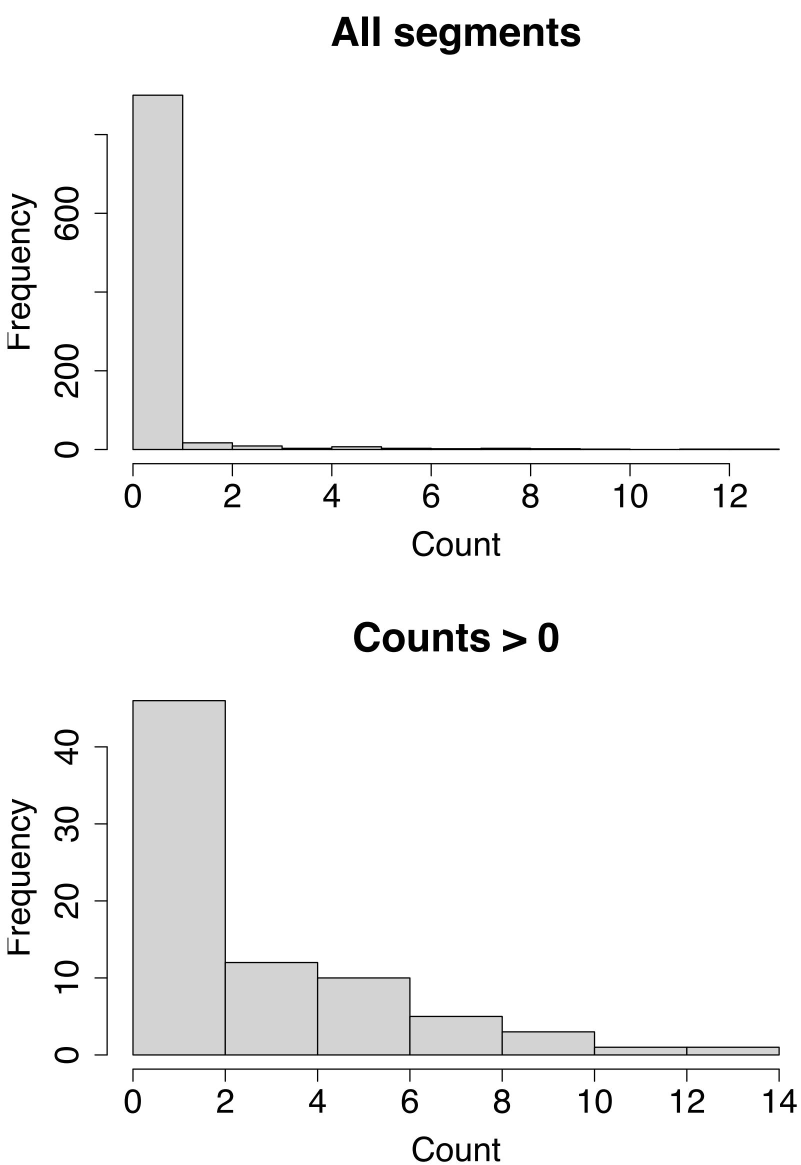
Now let's look at each bit...

Response

$$n_j = A_j \hat{p}_j \exp\left[\beta_0 + s(\text{y}_j) + s(\text{Depth}_j)\right] + \epsilon_j$$

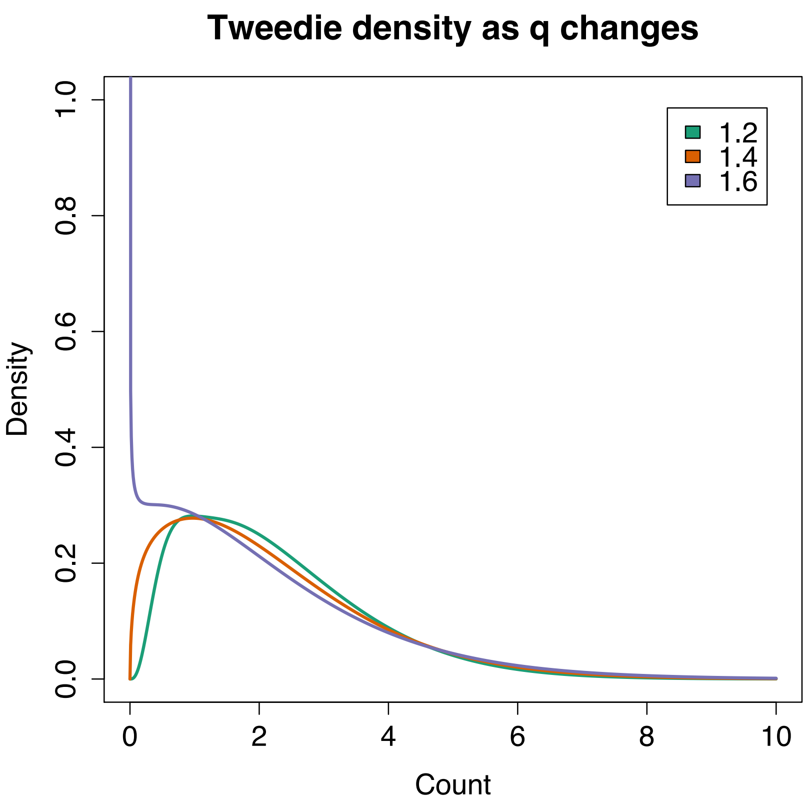
where $n_j \sim$ count distribution

Count distributions



- Response is a count
- Often, it's mostly zero
- mean $\neq$ variance
  - (Poisson isn't good at this)

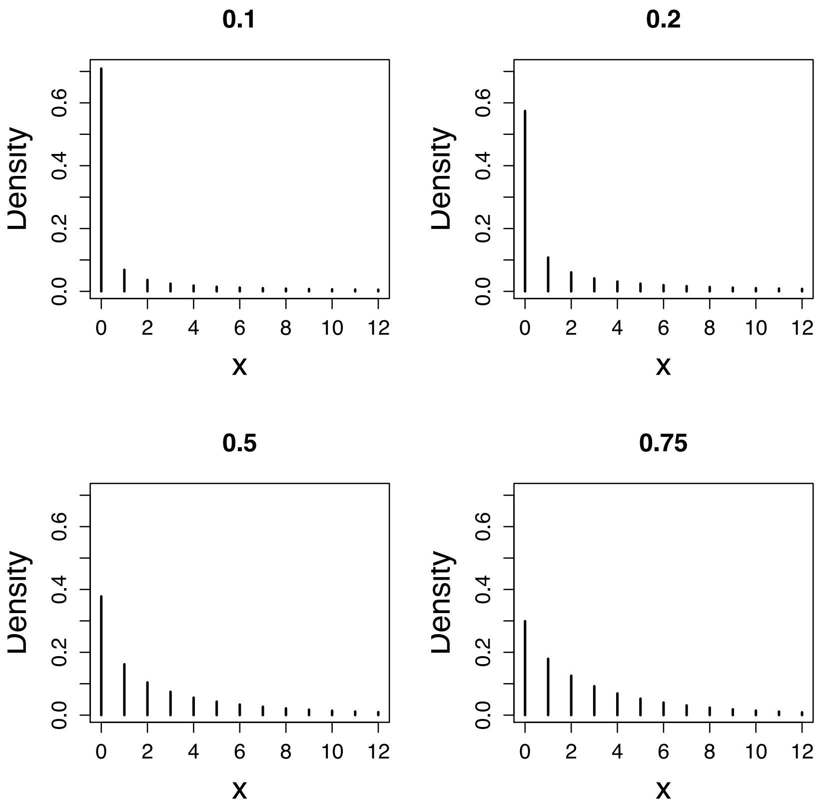
Tweedie distribution



(NB there is a point mass at zero not plotted)

- $\text{Var}\,(\text{count}) = \phi \mathbb{E}(\text{count})^q$
- Poisson is $q = 1$
- We estimate q and ϕ

Negative binomial distribution

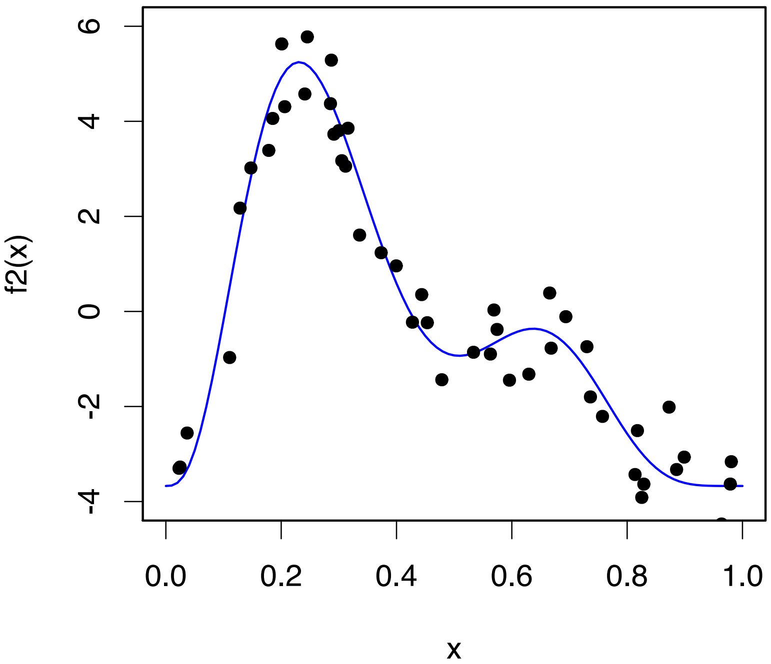


- $\mathrm{Var}\,(\text{count}) = \mathbb{E}(\text{count}) + \kappa\mathbb{E}(\text{count})^2$
- Estimate κ
- (Poisson: $\mathrm{Var}\,(\text{count}) = \mathbb{E}(\text{count})$)

Smooths

$$n_j = A_j \hat{p}_j \exp\left[\beta_0 + s(\mathrm{y}_j) + s(\mathrm{Depth}_j)\right] + \epsilon_j$$

What about these "s" things?



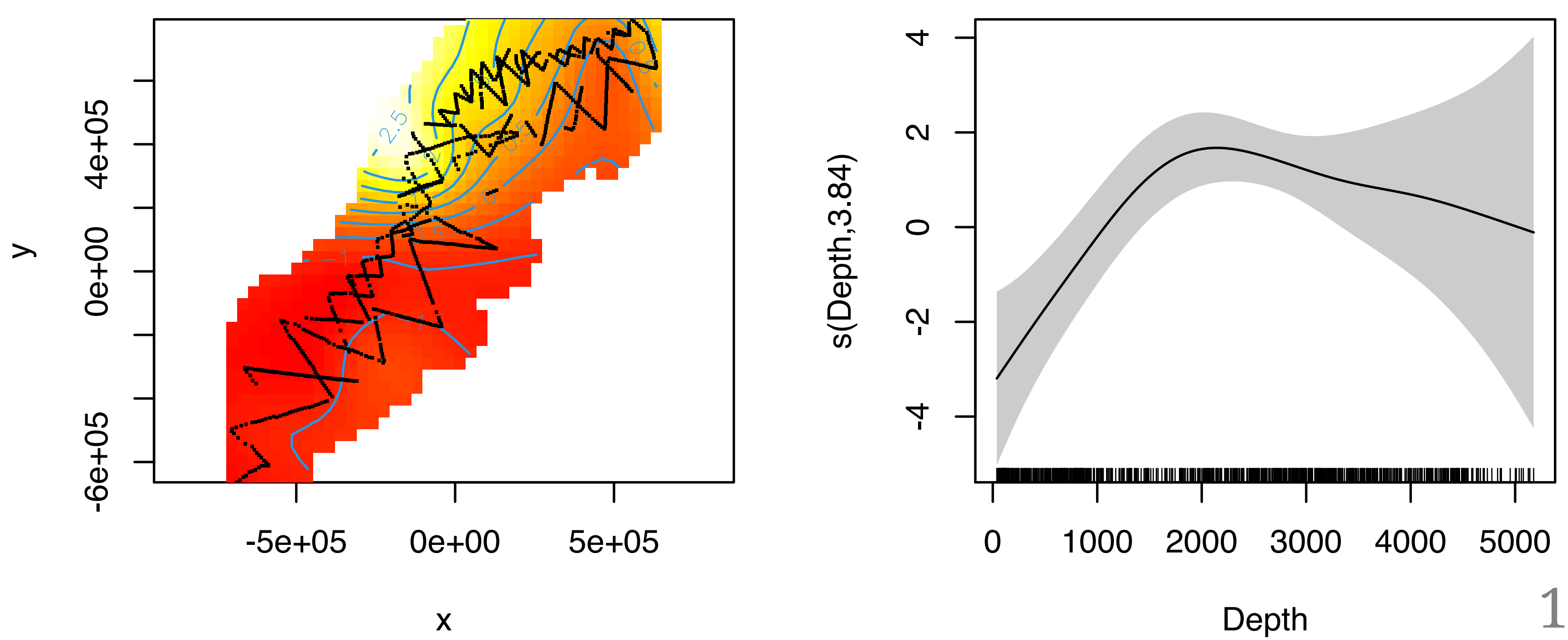
- *Think s*=**smooth**
- Want a line that is "close" to all the data
- Balance between interpolation and "fit"

What is smoothing?

Smoothing

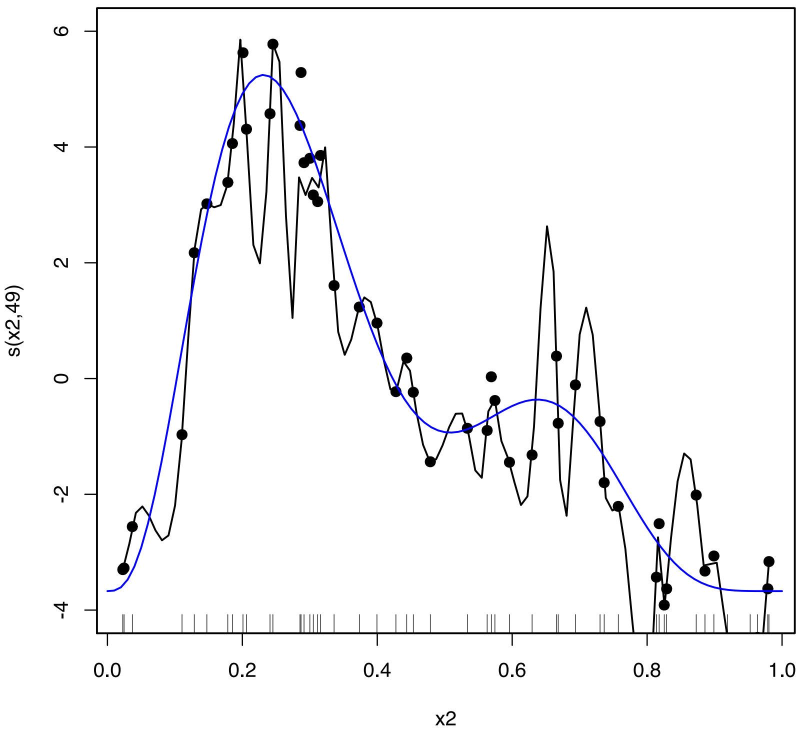
- We think underlying phenomenon is *smooth*
  - "Abundance is a smooth function of depth"
- 1, 2 or more dimensions

s(x,y,11.11)

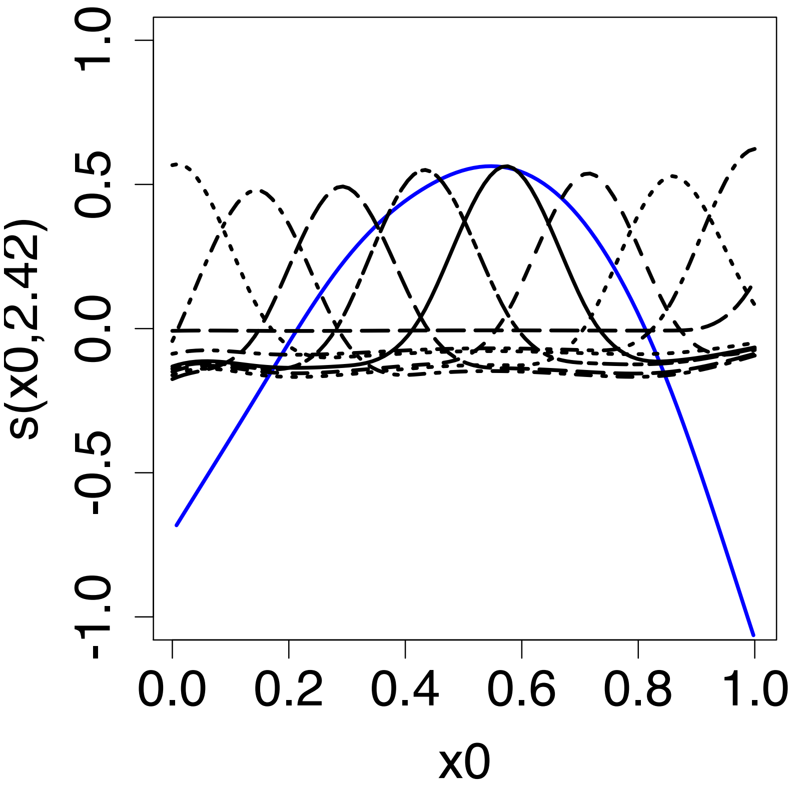


Estimating smooths

- We set:
  - "type": *bases* (made up of *basis functions*)
  - "maximum wigglyness": *basis size* (sometimes: dimension/complexity)
- Automatically estimate:
  - "how wiggly it needs to be": *smoothing parameter(s)*



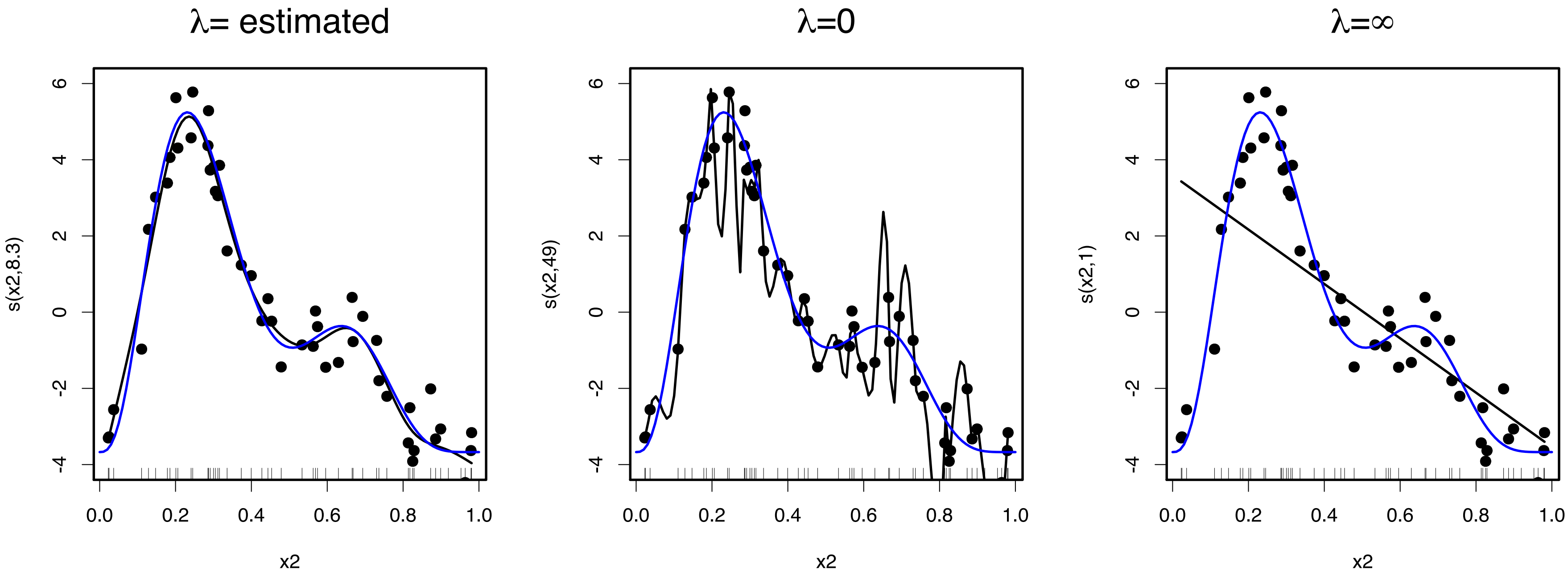
Splines



- Functions made of other, simpler functions
- **Basis functions** b_k, estimate β_k
- $s(x) = \sum_{k=1}^{K} \beta_k b_k(x)$

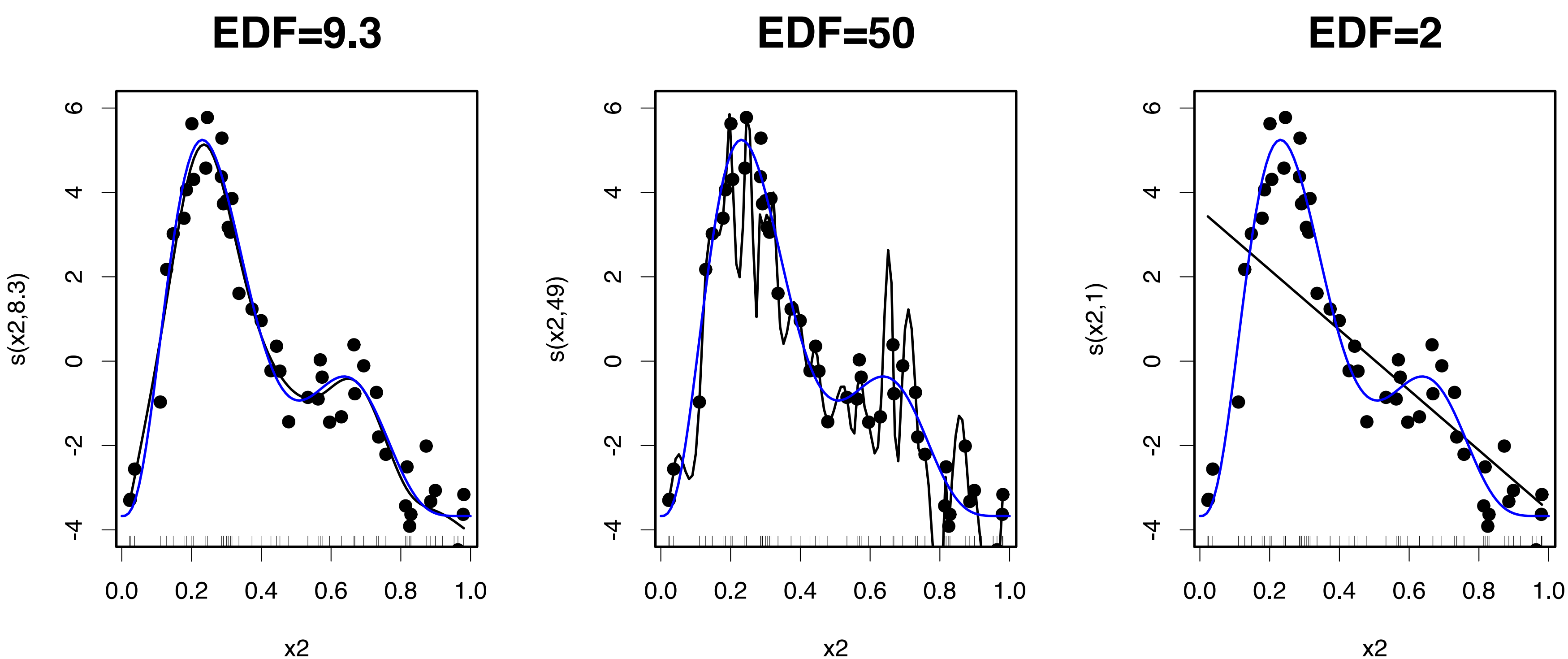
Measuring wigglyness

- Visually:
  - Lots of wiggles ⇒ *not smooth*
  - Straight line ⇒ *very smooth*

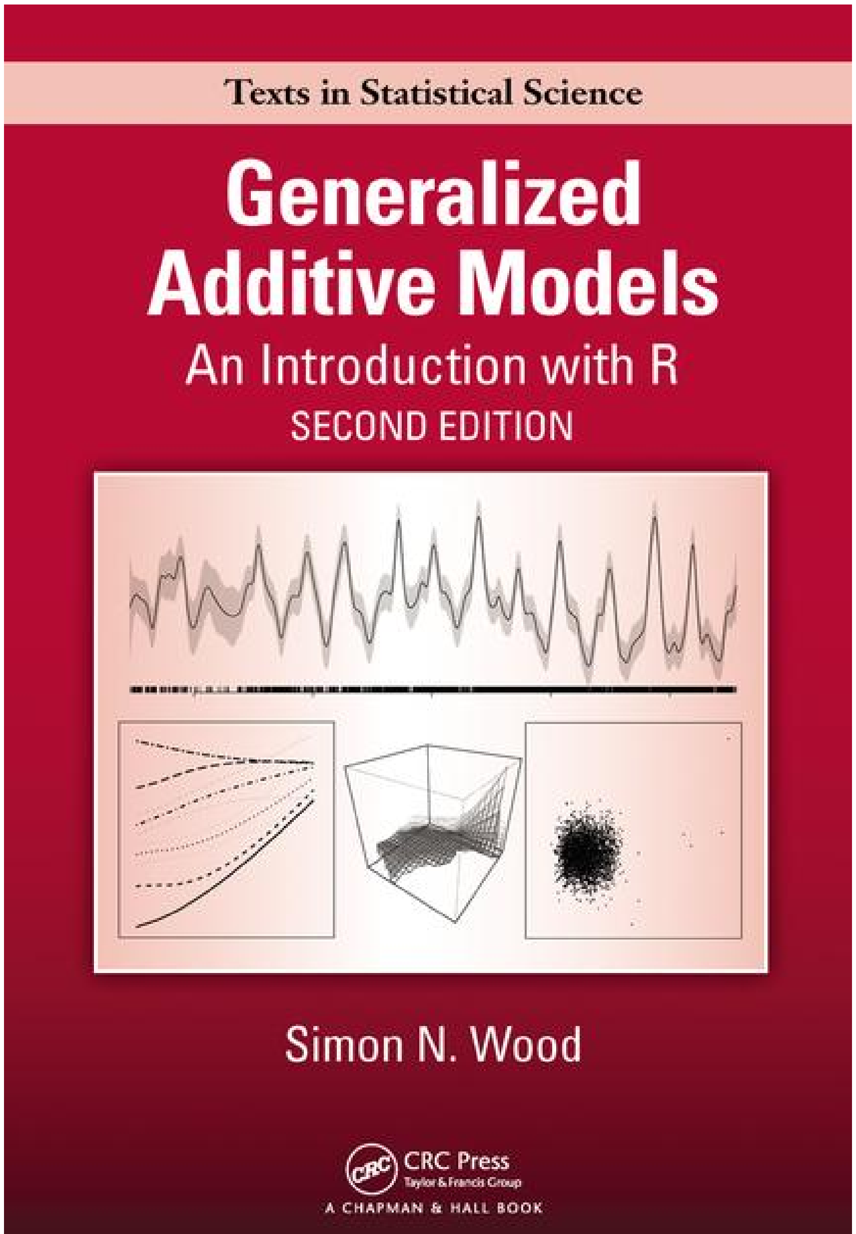


How wiggly are things?

- Set **basis complexity** or "size" k
- Fitted smooths have **effective degrees of freedom** (EDF)
- Set k "large enough"



Getting more out of GAMs



- I can't teach you all of GAMs in 1 week
- Good intro book
- (also a good textbook on GLMs and GLMMs)
- Quite technical in places
- More resources on course website

Fitting GAMs using dsm

Translating maths into R

$$n_j = A_j \hat{p}_j \exp\left[\beta_0 + s(\mathrm{y}_j)\right] + \epsilon_j$$

where ϵ_j are some errors, $\quad n_j \sim$ count distribution

- inside the link: `formula=count ~ s(y)`
- response distribution: `family=nb()` or `family=tw()`
- detectability: `ddf.obj=df_hr`
- offset, data: `segment.data=segs, observation.data=obs`

Your first DSM

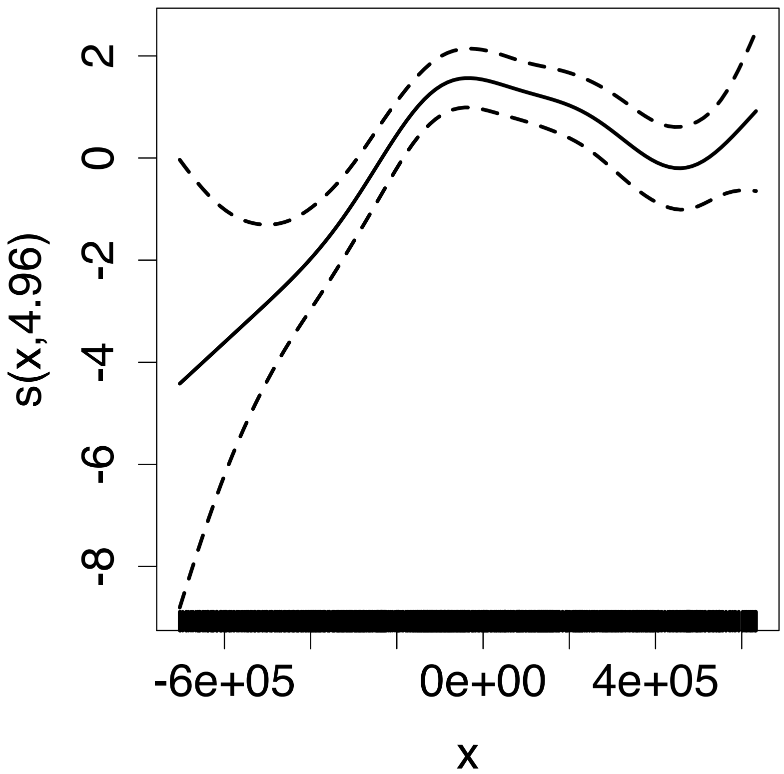
```
library(dsm)
dsm_x_tw <- dsm(count~s(x), ddf.obj=df,
                segment.data=segs, observation.data=obs,
                family=tw())
```

`dsm` is based on `mgcv` by Simon Wood

summary(dsm_x_tw)

```
##
## Family: Tweedie(p=1.326)
## Link function: log
##
## Formula:
## count ~ s(x) + offset(off.set)
##
## Parametric coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept) -19.8115     0.2277  -87.01   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Approximate significance of smooth terms:
##         edf Ref.df     F  p-value
## s(x) 4.962  6.047 6.403 1.07e-06 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## R-sq.(adj) =  0.0283   Deviance explained = 17.9%
## -REML = 409.94  Scale est. = 6.0413    n = 949
```

Plotting



- `plot(dsm_x_tw)`
- Dashed lines indicate +/- 2 standard errors
- Rug plot
- On the link scale
- EDF on y axis

Adding a term

- Just use +

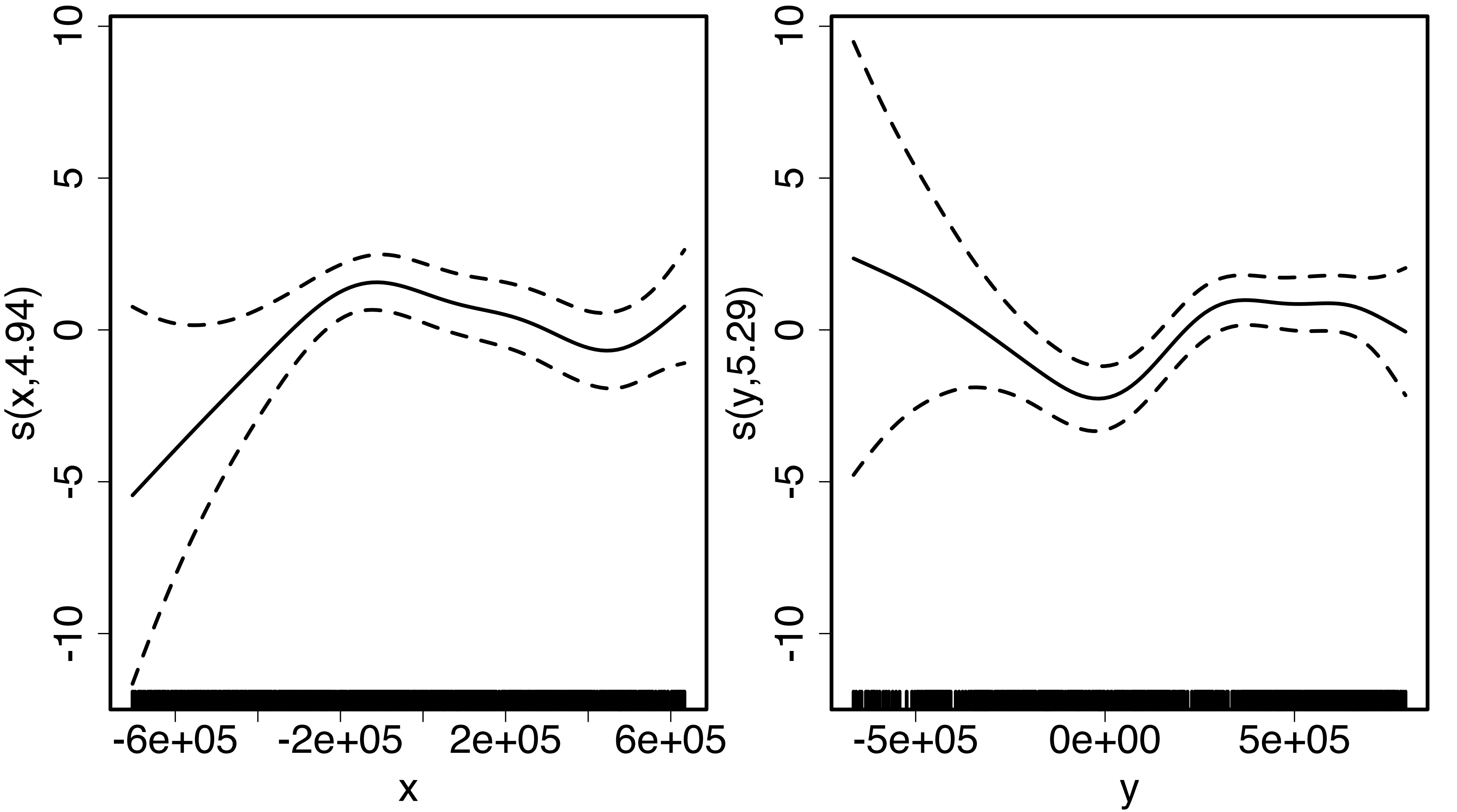
```
dsm_xy_tw <- dsm(count ~ s(x) + s(y),
                 ddf.obj=df,
                 segment.data=segs,
                 observation.data=obs,
                 family=tw())
```

summary(dsm_xy_tw)

```
##
## Family: Tweedie(p=1.306)
## Link function: log
##
## Formula:
## count ~ s(x) + s(y) + offset(off.set)
##
## Parametric coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept) -20.0908     0.2381  -84.39   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Approximate significance of smooth terms:
##        edf Ref.df     F  p-value
## s(x) 4.943  6.057 3.224 0.004239 **
## s(y) 5.293  6.419 4.034 0.000322 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## R-sq.(adj) =  0.0678   Deviance explained = 27.4%
## -REML = 399.84  Scale est. = 5.3157    n = 949
```

Plotting

```
plot(dsm_xy_tw, pages=1)
```



Bivariate terms

- Assumed an additive structure
- No interaction
- We can specify `s(x,y)` (and `s(x,y,z,...)`)

Bivariate spatial term

```
dsm_xyb_tw <- dsm(count ~ s(x, y),
                  ddf.obj=df,
                  segment.data=segs,
                  observation.data=obs,
                  family=tw())
```

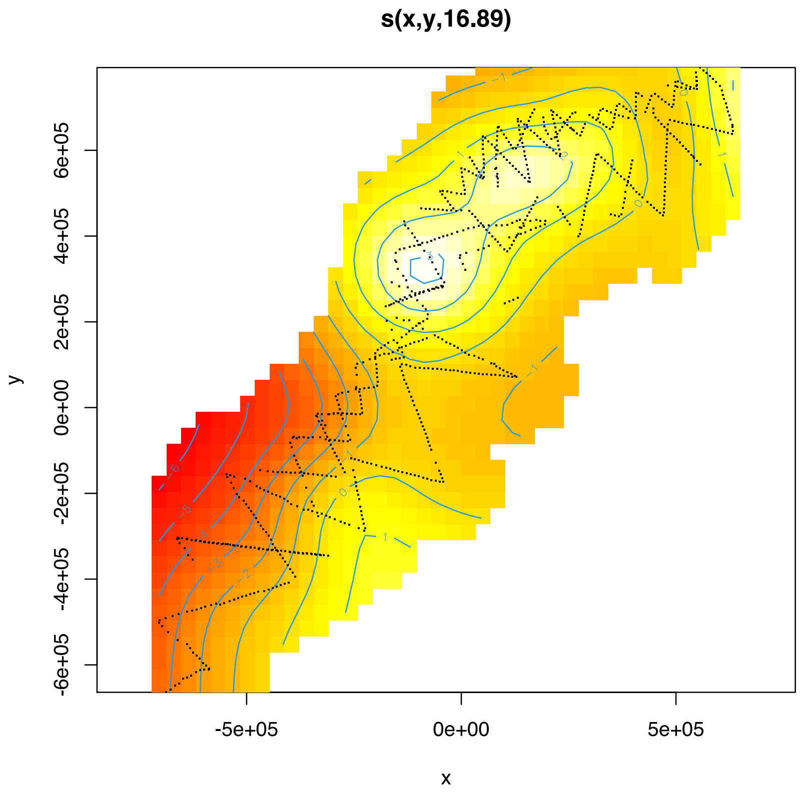
summary(dsm_xyb_tw)

```
##
## Family: Tweedie(p=1.29)
## Link function: log
##
## Formula:
## count ~ s(x, y) + offset(off.set)
##
## Parametric coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept) -20.2745     0.2477  -81.85   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Approximate significance of smooth terms:
##          edf Ref.df     F  p-value
## s(x,y) 16.89  21.12 4.333 3.73e-10 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## R-sq.(adj) =  0.102   Deviance explained = 34.7%
## -REML = 394.86  Scale est. = 4.8248    n = 949
```

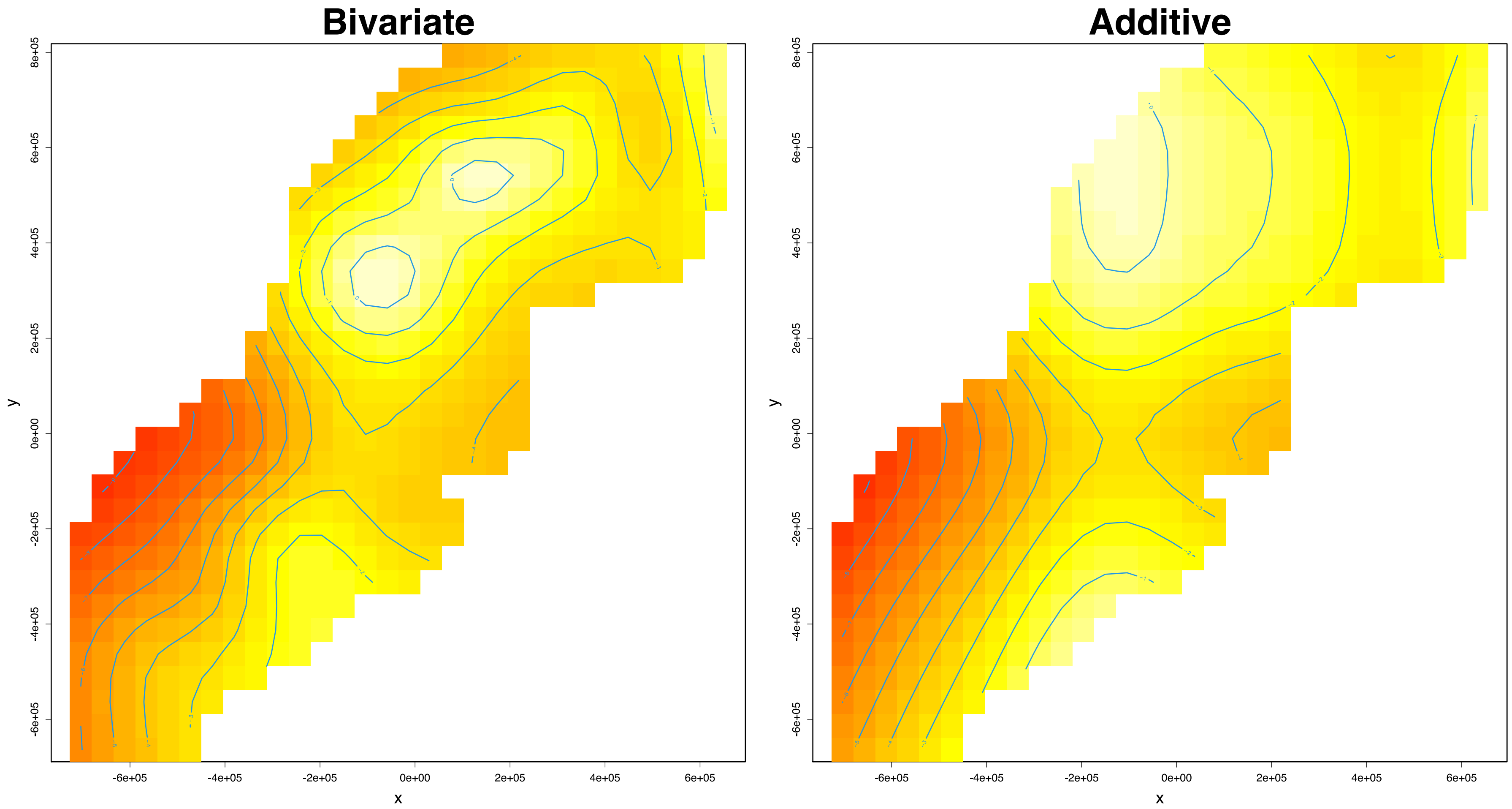
Plotting

```
plot(dsm_xyb_tw, select=1,
     scheme=2, asp=1)
```

- On link scale
- `scheme=2` makes heatmap
- (set `too.far` to exclude points far from data)



Comparing bivariate and additive models



Let's have a go...