Three ways to think about detectability in distance sampling
1. The detection function, $g(x)$

$g(x) = \text{probability of detecting an animal, given that it is at distance } x \text{ from the line}$

We assume $g(0) = 1$

Note: histogram bars scaled

\[ \hat{P}_a = \frac{\text{area under curve}}{\text{area under rectangle}} = \frac{\int_0^w \hat{g}(x)dx}{1 \times w} \]
2. Effective strip (half) width, $\mu$

- Instead of a line transect out to $w$, where proportion $P_a$ objects are seen, think of a strip transect out to some distance $\mu$.

The ESW, $\mu$, is the distance at which as many objects are seen beyond $\mu$ as are missed within $\mu$.

**Line transect out to $w$**

\[ \hat{N} = \frac{nA}{2wL \hat{P}_a} \]

**Strip transect out to $\mu$**

\[ \hat{N} = \frac{nA}{2\mu L} \]

\[ \hat{P}_a = \frac{\text{area under curve}}{\text{area under rectangle}} = \frac{\int_{0}^{w} \hat{g}(x)dx}{w} = \frac{\hat{\mu}}{w} \]
3. The probability density function, \( f(x) \)

\[ f(x)dx = \text{probability of observing an animal between distance } x \text{ and } x+dx, \text{ given it was observed somewhere in } (0,w) \]

\( f(x) \) is called the probability density function (pdf) of the observed distances

Because observations are between 0 and \( w \), the area under \( f(x) \) is 1.0

\[
\int_{0}^{w} f(x)dx = 1
\]

Histogram bars are scaled so that area under histogram is 1.

Area under \( f(x) \) is 1
Why is $f(x)$ useful?

1. Useful for point transects, as it gives the expected distribution of detection distances

True distribution of animals

Detection function, $g(x)$

Observed distribution, $f(x)$

Line transect

Point transect

see lecture on point transects
Why is $f(x)$ useful?

2. Gives another way to estimate $P_a$

Lots of statistical machinery to fit pdfs, so this is the way Distance does it.

\[
\hat{P}_a = \frac{\text{area under curve}}{\text{area under rectangle}} = \frac{1}{\hat{f}(0)w}
\]

\[
\hat{N} = \frac{nA}{2wL\hat{P}_a} = \frac{nA}{2wL\left(\frac{1}{\hat{f}(0)w}\right)} = \frac{nA\hat{f}(0)}{2L}
\]

Question: How are $f(0)$ and $\mu$ related?
Three ways to think about line transects

1. Proportion seen or average probability of detection in covered region, $P_a$

$$\hat{N} = \frac{nA}{2wL\hat{P}_a} \quad \hat{D} = \frac{n}{2wL\hat{P}_a}$$

2. Effective strip (half-)width, ESW, $\mu$.

$$\hat{N} = \frac{nA}{2\hat{\mu}L} \quad \hat{D} = \frac{n}{2\hat{\mu}L}$$

3. Pdf of observed distances, $f(x)$, evaluated at 0 distance $f(0) = \frac{1}{\mu}$

$$\hat{N} = \frac{n\hat{f}(0)A}{2L} \quad \hat{D} = \frac{n\hat{f}(0)}{2L}$$
Notation – line transects

Known constants and data:

- $k$ = number of lines
- $l_j$ = length of $j^{th}$ line, $j=1,...,k$
- $L = \Sigma l_j$ = total line length
- $n$ = number of animals or clusters detected
- $x_i$ = distance of $i^{th}$ detected animal or cluster from the line, $i=1,...,n$
- $w$ = truncation distance for $x$
- $A$ = size of region of interest
- $a = \text{area of “covered” region } = 2wL$
- $s_i = \text{size of } i^{th} \text{ detected cluster, } i=1,...,n$
Notation – line transects

Parameters and functions:

\( N \) = population size / abundance of animals

\( N_s \) = abundance of clusters

\( D \) = density = animals per unit area = \( N/A \)

\( D_s \) = density of clusters

\( g(x) \) = detection function

\( f(x) \) = probability density function (pdf) of observed distances

\( f(0) = f(x) \) evaluated at 0 distance

\( \mu \) = effective strip (half-)width

\( P_\alpha \) = probability of detecting an animal or cluster given it is in the covered area \( \alpha \)

\( E(s) \) = mean size of clusters in the population