Assessment of model performance

- Likelihood
- AIC
- Absolute measures of model fit
  Chi-squared test
  Q-Q plots
  Kolmogorov-Smirnov and Cramér-von Mises tests

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Likelihood

$f(x)$ = probability density function of x

$f(x)\,dx$ = Pr (animal was between x and $x+dx$ from the line, given it was detected between 0 and w) for small dx

When distances are exact, the **likelihood** is given by

$$L = \prod_{i=1}^{n} f(x_i) = f(x_1) \times f(x_2) \times \ldots \times f(x_n)$$

x_i = distance of i^{th} detected animal from the line.

We fit $f(x)$ by finding the values for the parameters of $f(x)$ (or equivalently $g(x)$) that maximize L (or $\log_e(L)$).

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Akaike's Information Criterion

$$AIC = -2\log_e(L) + 2q$$

L is the maximized likelihood (evaluated at the maximum likelihood estimates of the model parameters)

and q is the number of parameters in the model.

- Models need not be special cases of one another
- Select the model with smallest AIC
- Gives a relative measure of fit

Limitations of AIC

Cannot be used to select between models when:

- sample size *n* differs
- truncation distance *w* differs
- data are grouped, and cut points differ
- data are grouped in one analysis and ungrouped in the other

Goodness-of-Fit

- Chi-squared test for grouped (interval) data; if data are exact, we must specify interval cut points for this test
- Q-Q plots and related tests for exact data

Chi-squared tests

Define u distance intervals, with n_i detections in interval i, i = 1, ..., u.

Then
$$\chi^2 = \sum_{i=1}^{u} \frac{(n_i - n\hat{\pi}_i)^2}{n\hat{\pi}_i}$$

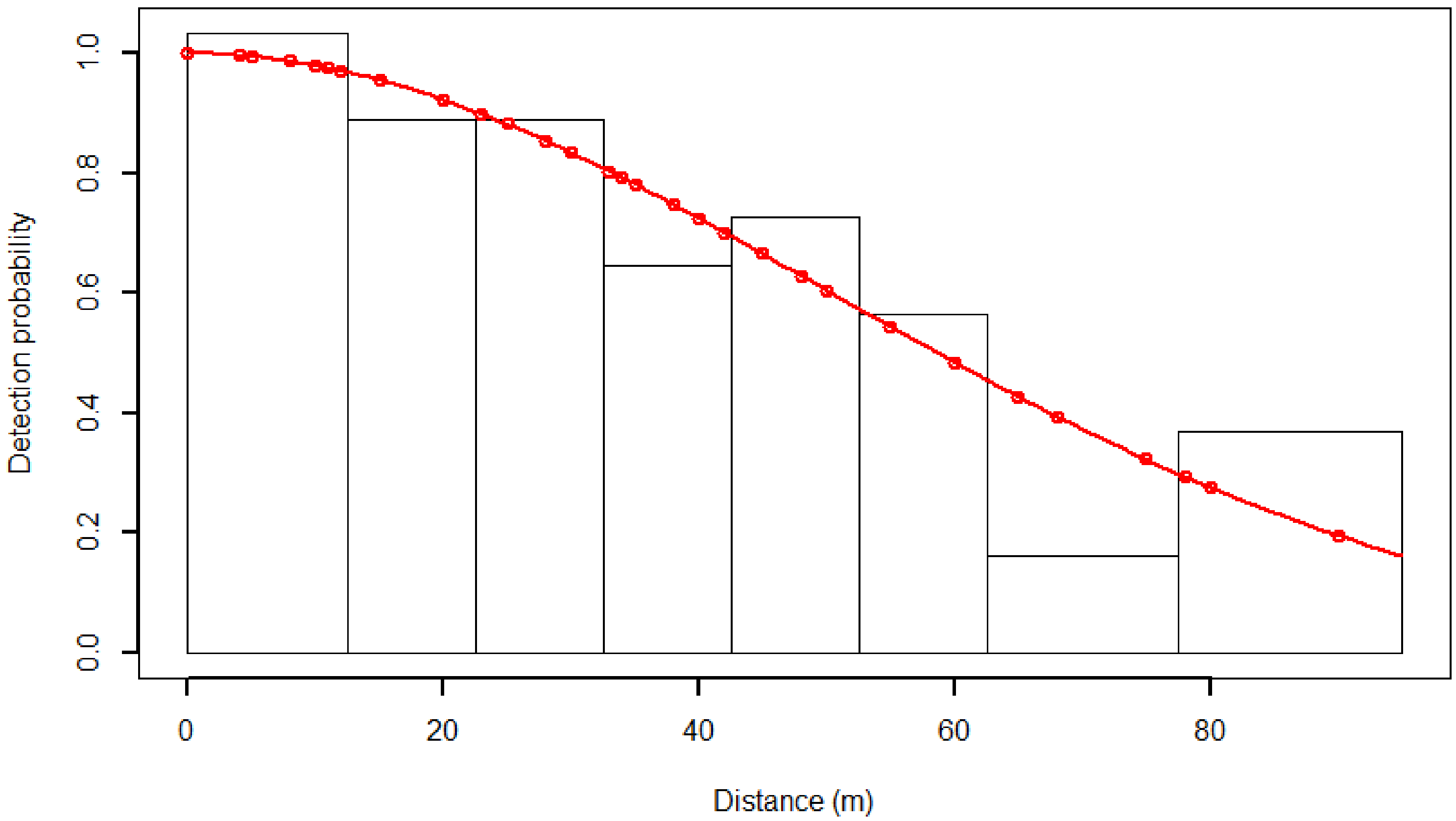
where $n = \sum_i n_i$

and $\hat{\pi}_i$ is the proportion of the area under the estimated pdf, $\hat{f}(x)$, that lies in interval i.

If the model is 'correct': $\chi^2 \sim \chi^2_{u-q-1}$

q = no. of parameters

Chaffinch line transect data



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χ^2 goodness-of-fit test

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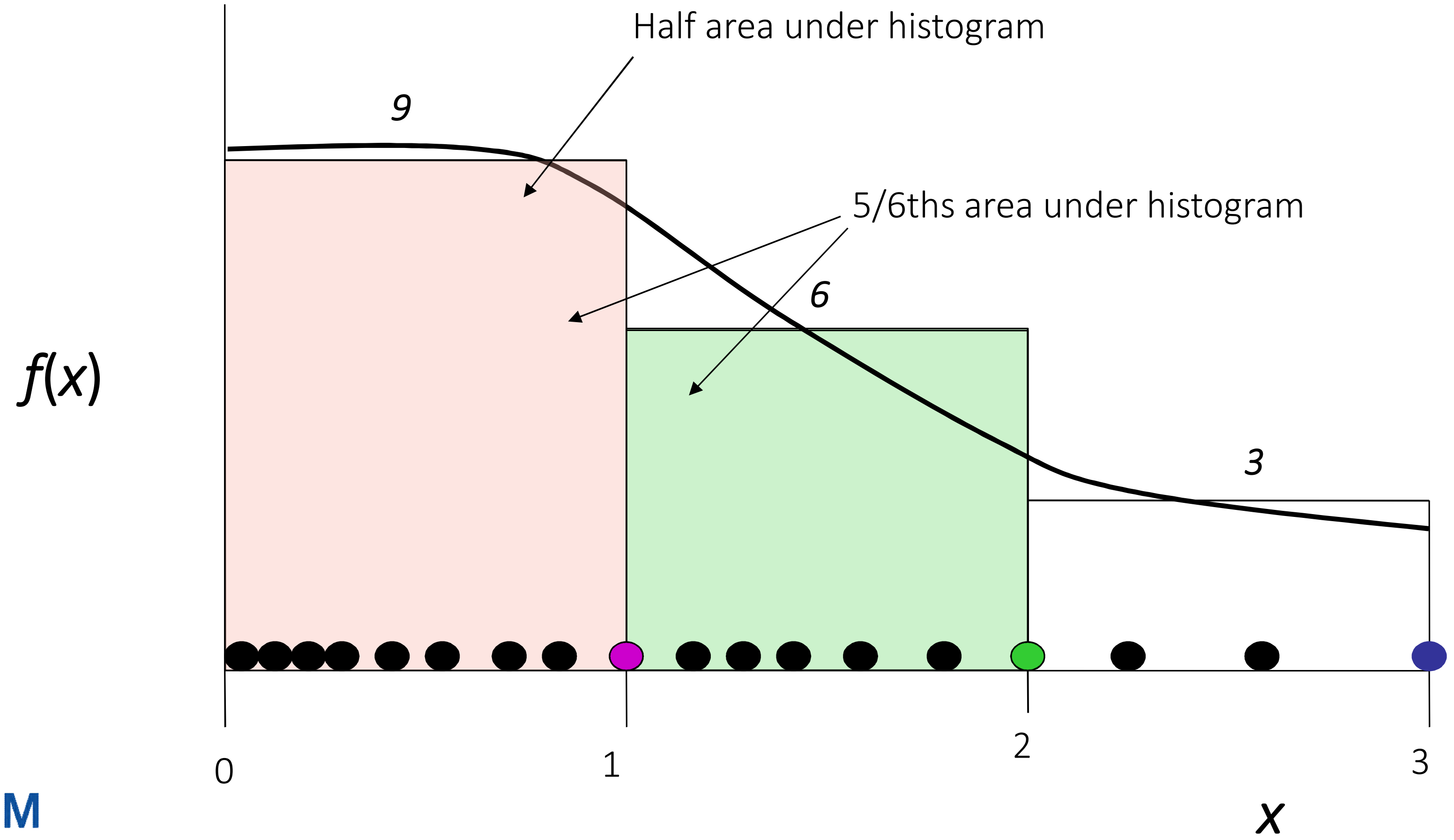
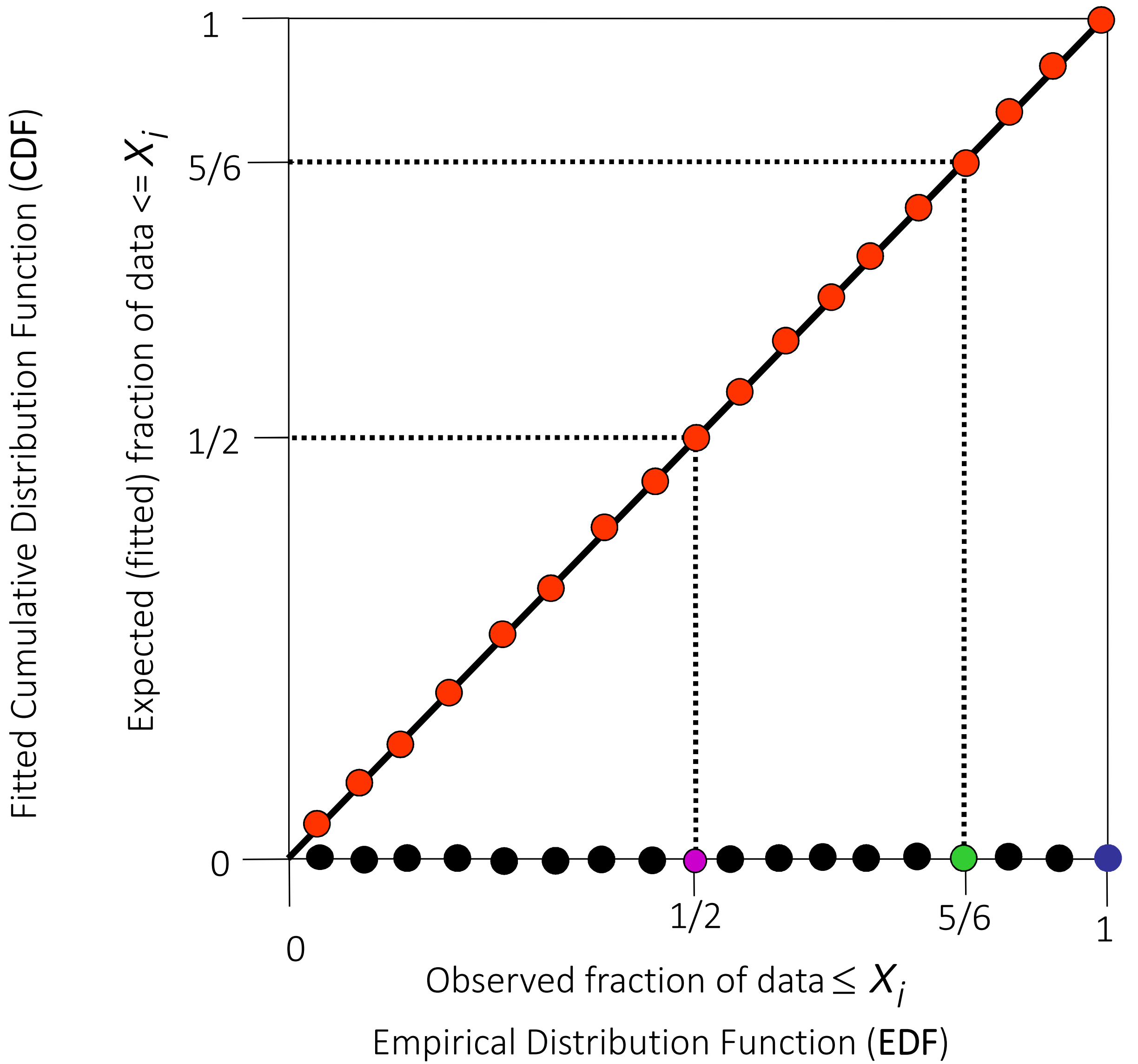
Goodness of fit results for ddf object

Chi-square tests
             [0,12.5] (12.5,22.5] (22.5,32.5] (32.5,42.5]
Observed  16.00000000 11.00000000   11.000000   8.0000000
Expected  15.31832030 11.62653282   10.623975   9.3264854
Chisquare  0.03033539  0.03376272    0.013309   0.1886631
          (42.5,52.5] (52.5,62.5] (62.5,77.5] (77.5,95]     Total
Observed    9.0000000  7.00000000    3.000000  8.000000 73.000000
Expected    7.8658030  6.37326777    6.960224  4.905391 73.000000
Chisquare   0.1635437  0.06163138    2.253286  1.952261  4.696791

P = 0.58325 with 6 degrees of freedom
```

Q-Q Plots and Related Tests





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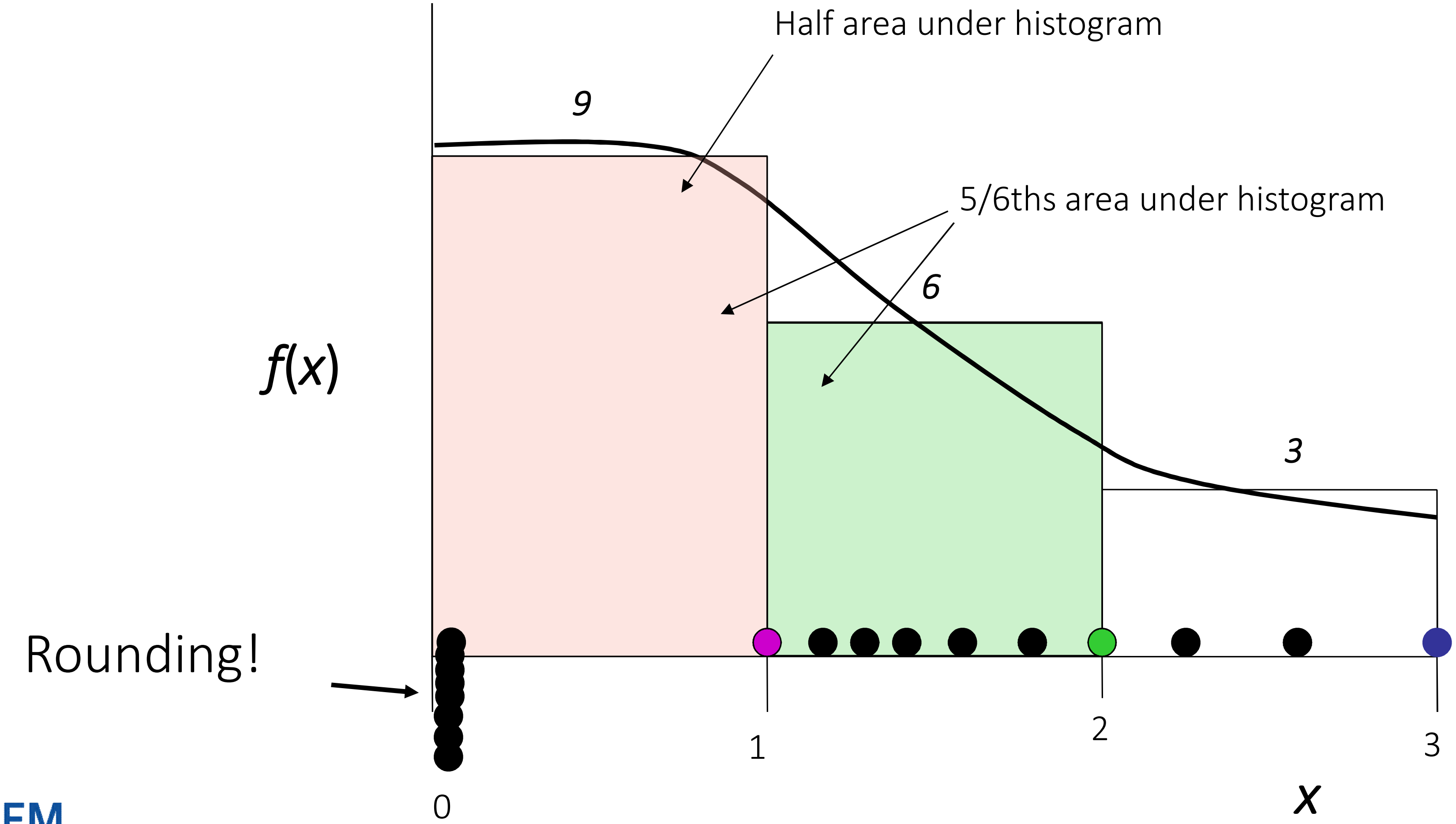
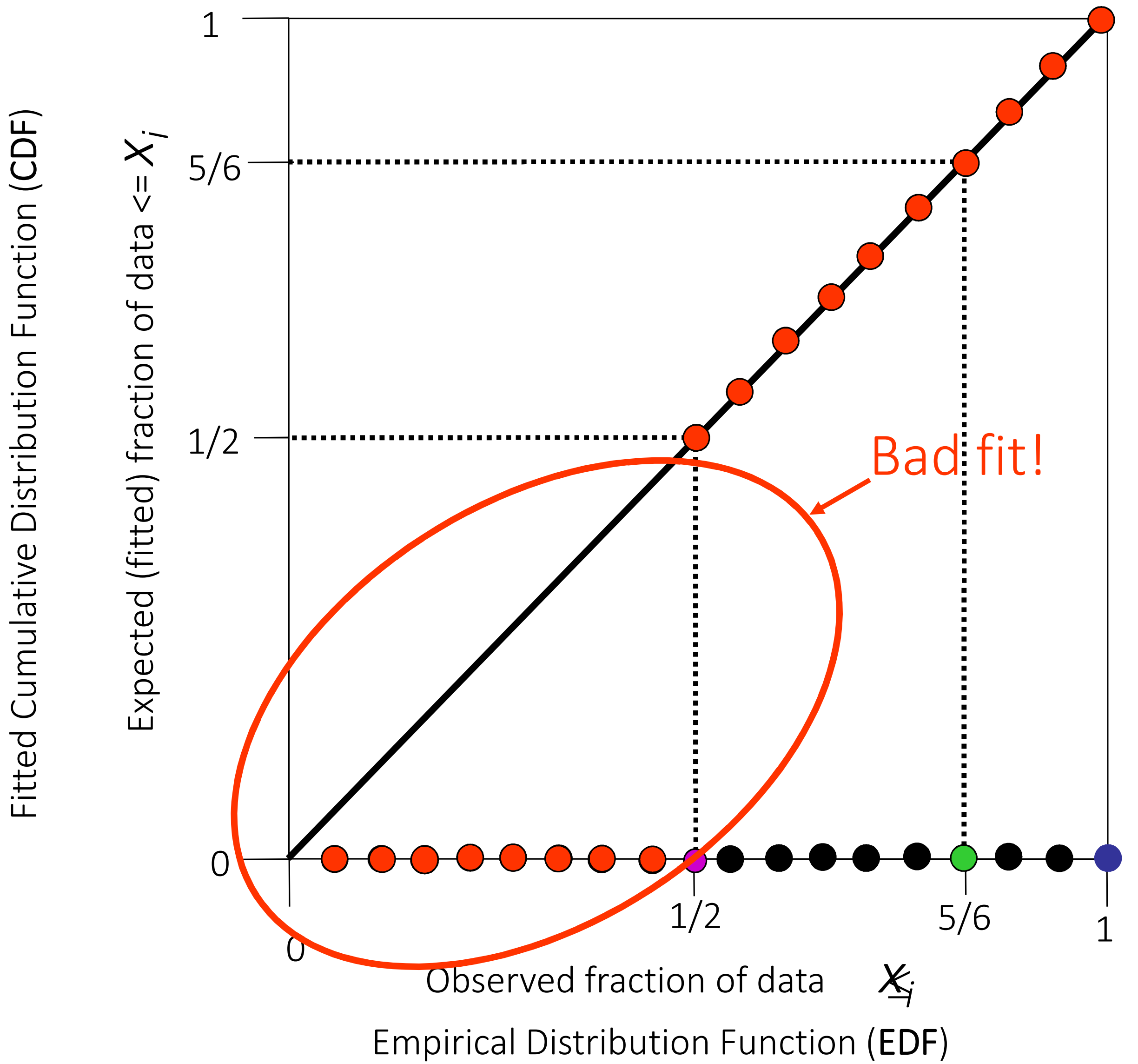
Example: Rounding to zero





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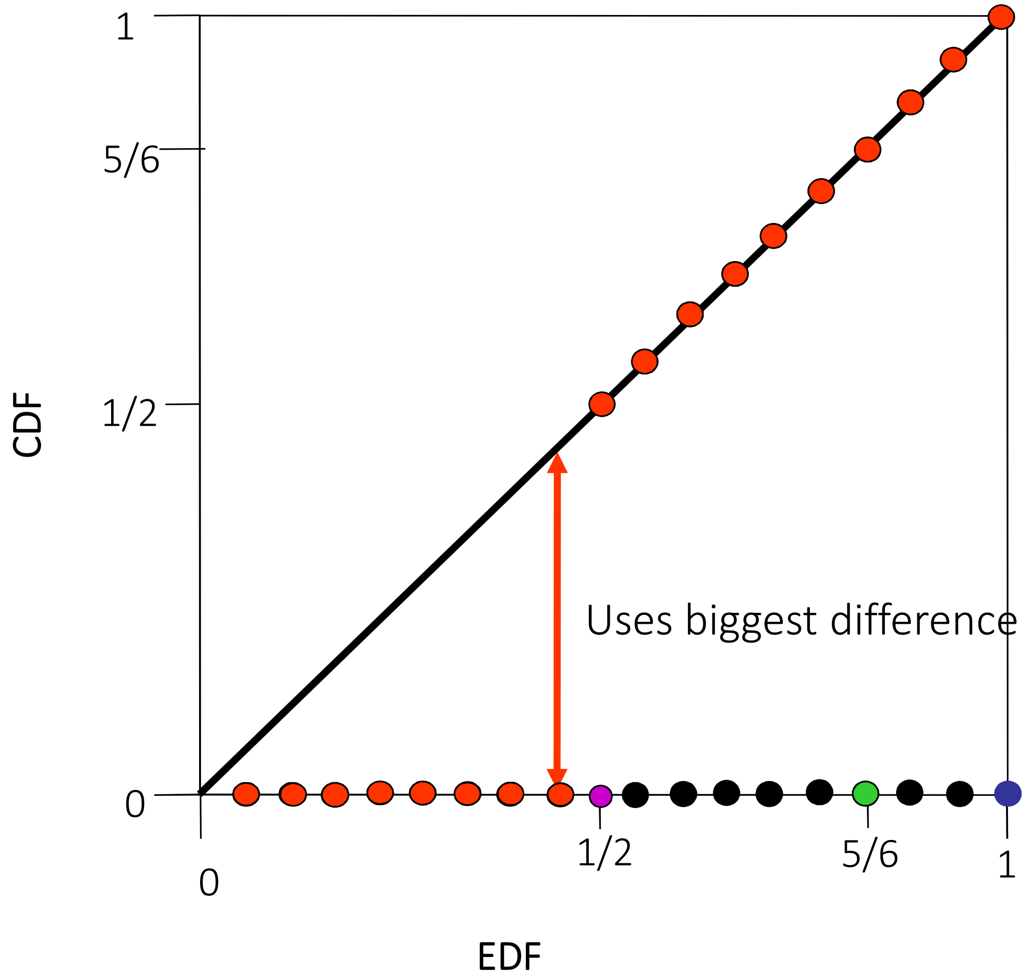
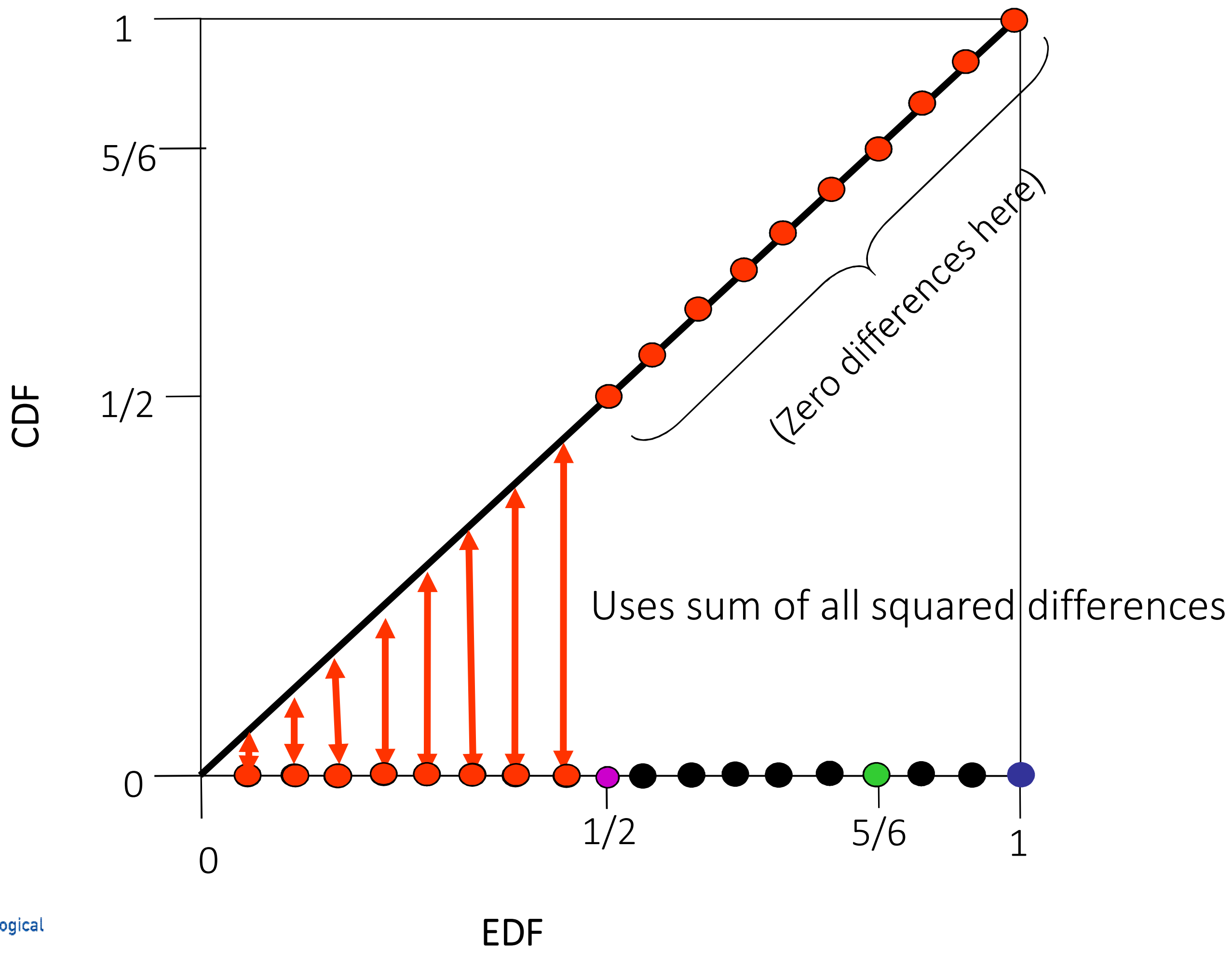
Kolmogorov-Smirnov test



Cramér-von Mises test



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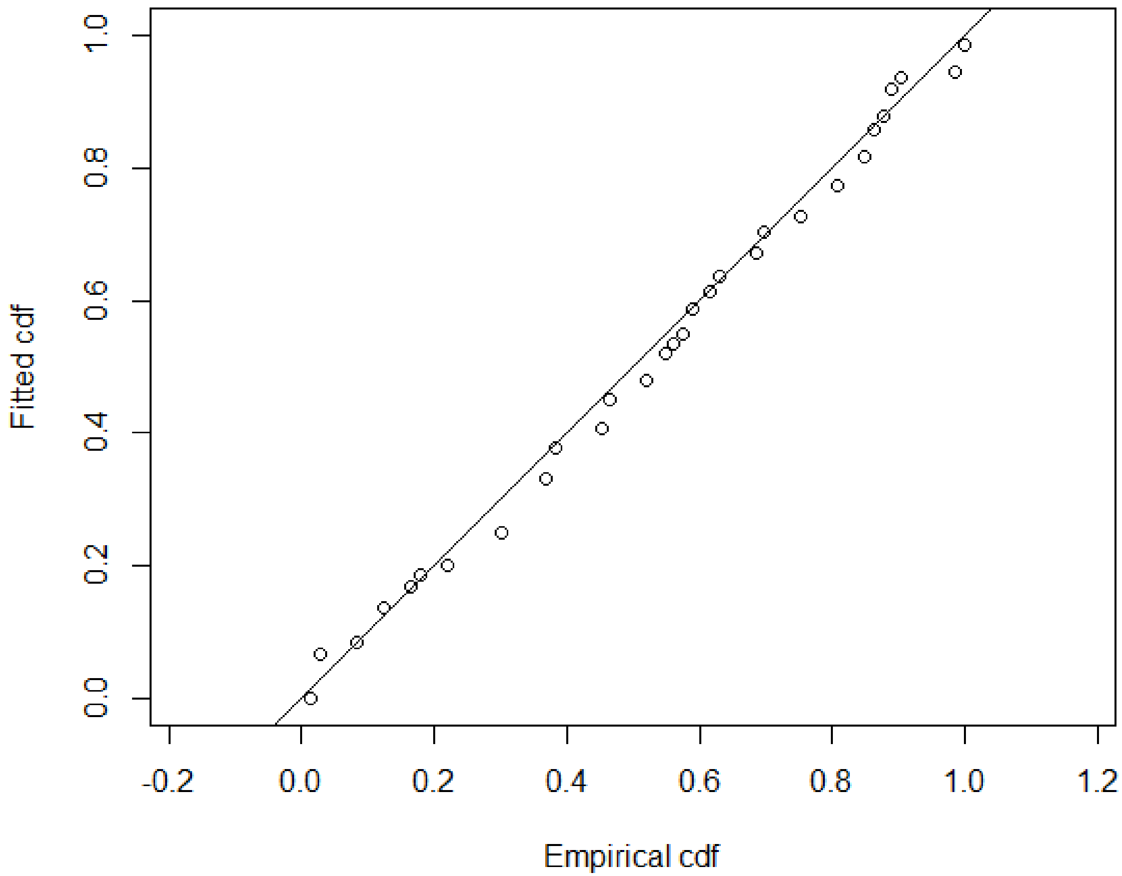
Chaffinch line transect Q-Q plot



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K-S test and Cramer-von Mises test

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Distance sampling Kolmogorov-Smirnov test
Test statistic = 0.0572767 p-value = 1
 (p-value calculated from 100/100 bootstraps)

Distance sampling Cramer-von Mises test (unweighted)
Test statistic = 0.0367951 p-value = 0.948916
```

Q-Q Plot Summary

- Q-Q plots show goodness-of-fit at “high resolution” – without requiring grouping into intervals
- Kolmogorov-Smirnov test and Cramér-von Mises test are goodness-of-fit tests that do not require grouping

Making Distance Sampling Work

- Assumptions and effect of violation
- Reliable distance sampling
- Pooling robustness
- Examples of imperfect data

Recap of distance sampling

There are two stages to estimating abundance

Stage 1: given n, how many objects are in the surveyed/covered region (of size a), N_a

Need to estimate P_a *(or* $f(0)$ *or ESW, etc.)*

$$\hat{N}_a = {}^{n}\!/_{\hat{P}_a}$$

Stage 2: given $\hat{N}_a$, how many objects are in study region (of size A), N

'Scale up' from what we see in the survey region to the whole study region

$$\hat{N} = \frac{\hat{N}_a}{{}^{a}\!/_{A}}$$

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Assumptions for estimating N_a (stage 1)

1. **Animals distributed independently of line or point**

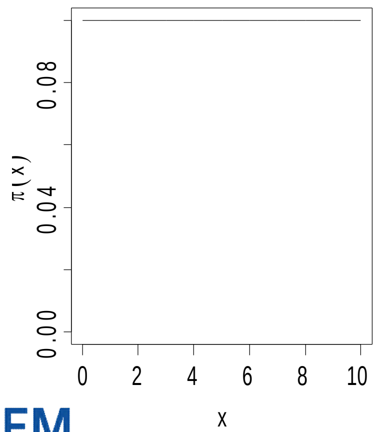
   This ensures the true distribution of animals with respect to the line or point is known

   Violated by non-random line/point placement

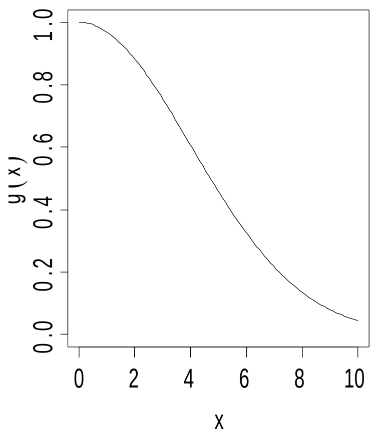
   Substantial violation can produce substantial bias (e.g. roadside counts)

   e.g. for line transects

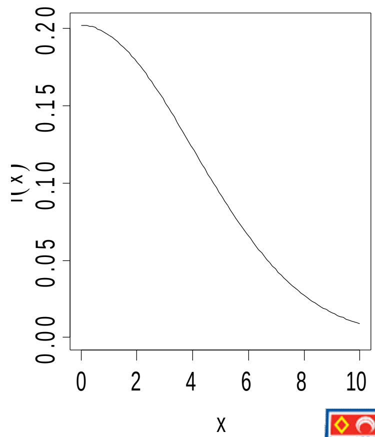
True distribution of animals



Detection function, $g(x)$



Observed distribution, $f(x)$



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Assumptions for estimating N_a (stage 1)

2. **All animals on the line or point are detected i.e. $g(0)=1$**

   It is a critical assumption - violation causes negative bias

   e.g. if $g(0)=0.8$, estimates of N are 80% of true N on average

Assumptions for estimating N_a (stage 1)

3. Observation process is a ‘snapshot’

Other ways to phrase this:

Observers are moving much faster than the animals

Animals do not move before they can be detected

Problems of independent/non-responsive movement

An animal moving independently of the observer (compared to moving in response to the observer) produces positive bias; size of bias depends on relative rate of movement of observer and animal, and type of survey.

Point transect methods in particular need to use ‘snapshot’ method.

Assumptions for estimating N_a (stage 1)

3. Observation process is a 'snapshot' (continued…)

Problems of responsive movement

Responsive movement can cause large bias

It can occur ***within*** *a single line/point or* ***between*** *lines/points*

If animals are 'driven' from one line/point to the next ahead of the observer, positive bias will result.

Note: movement independent of observer outwith 'snapshot' is fine – in this case, the same animal can be detected on multiple lines/transects

Assumptions for estimating N_a (stage 1)

4. **Distances are measured accurately**

   Random errors cause bias.

   Bias is generally small for line transect estimators,

   Can be large for point transect estimators.

   Both are sensitive to systematic bias and to rounding to 0 distance (or angle).

   Can use grouped data collection.

5. **Detections are independent**

   Violation has little effect. (Model selection methods for $g(x)$, such as AIC, are somewhat affected)

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Assumptions for estimating N given N_a (stage 2)

1. Lines or points are located according to a survey design with appropriate randomization

   We use properties of the survey design to extrapolate from the surveyed/covered region to the study region ('design-based')

   Non-random survey design means density in surveyed/covered region may not be representative of density in study region. Also variance may be biased.

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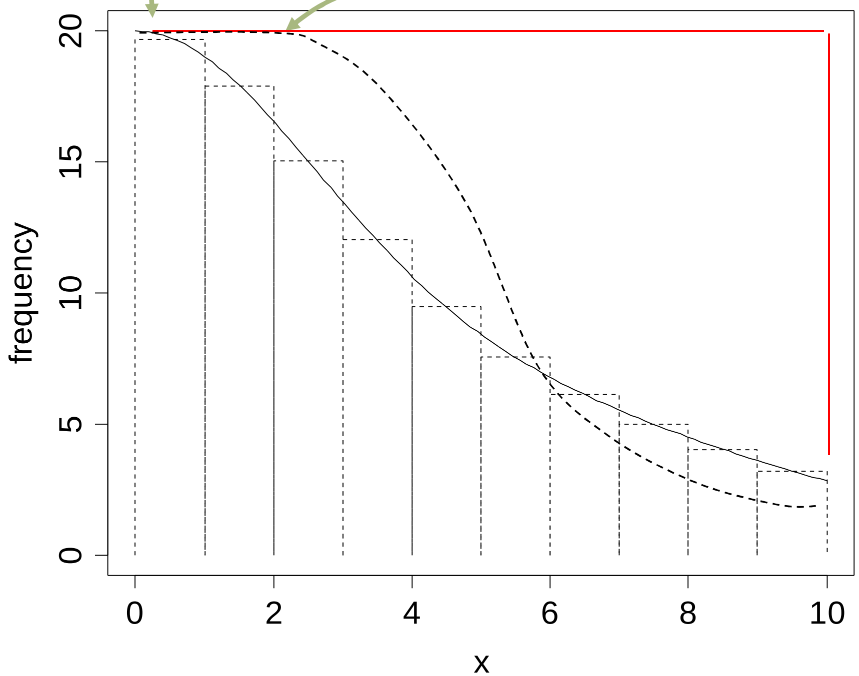
Reliable distance sampling (1)

1. Reliable estimation of P_a (or $f(0)$ or ESW, etc.)

In addition to the assumptions, we would like:

SHAPE CRITERION
Detection function should have a 'shoulder' (i.e. $g'(0)=0$)

Data that have a wide shoulder are preferable



A wide shoulder makes it easier to estimate area under rectangle (or $f(0)$, etc.)

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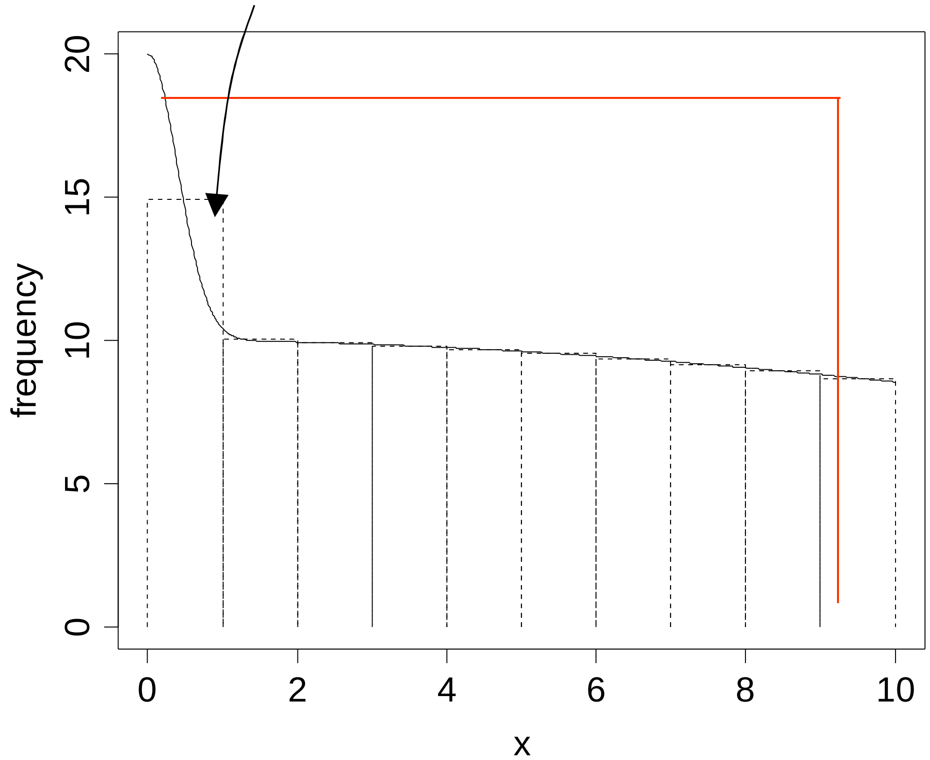
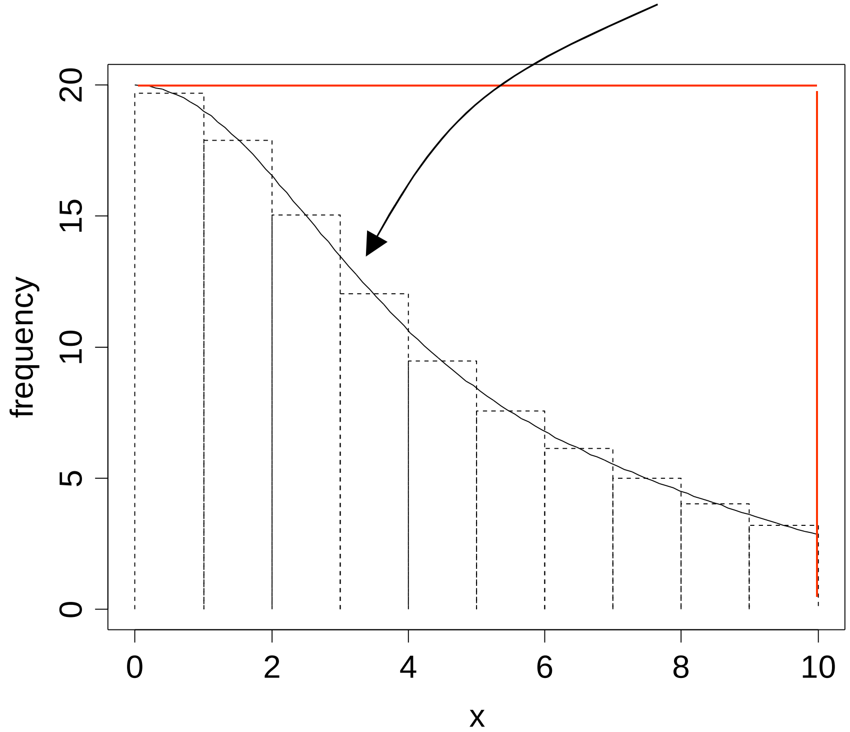
(1) Reliable estimation of P_a

Good field methods will avoid a ‘spike’ like this



Avoid a) rounding distances (and angles) to zero,
b) ‘guarding the trackline’

(1) Reliable estimation of P_a (cont.)



Sample size of observations (~60-80)

- less for detection functions with ‘easy’ shapes
- more for point transects and ‘difficult shapes’.

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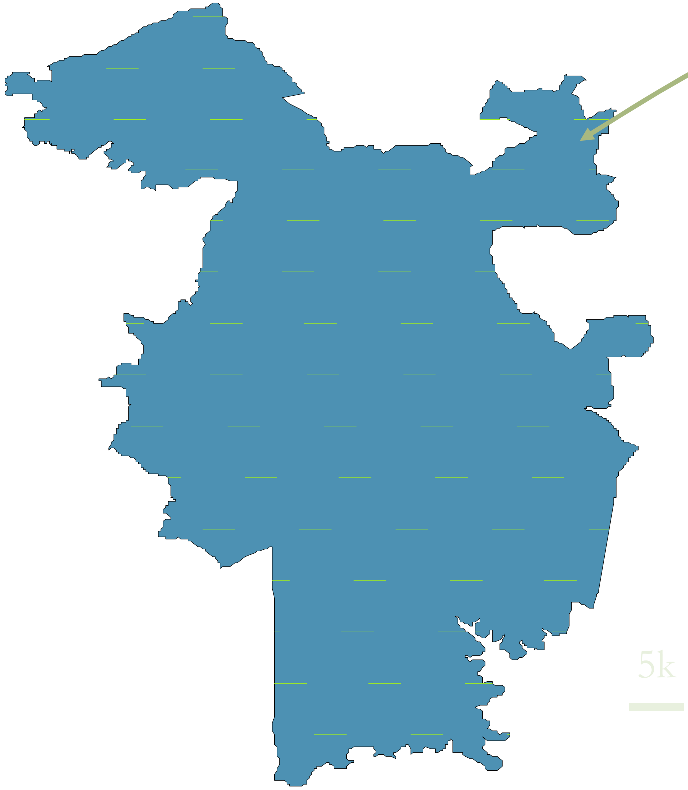
Reliable distance sampling (2)

2. Reliable estimation of N from N_a

In addition to the assumption of randomized design, we would like a 'large' sample of lines or points (20 or more), evenly distributed through the study region

see lecture on survey design



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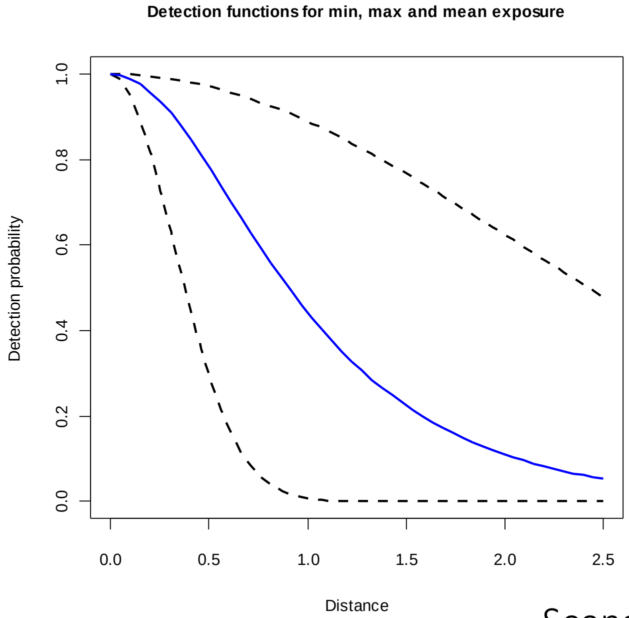
Pooling robustness

Individuals can have quite different detection functions, but this produces little bias (up to a point!)

‘Pooling robustness’ = robust to pooling of multiple detection functions

e.g. Simulation study (unpublished!) Truth = 1000 animals

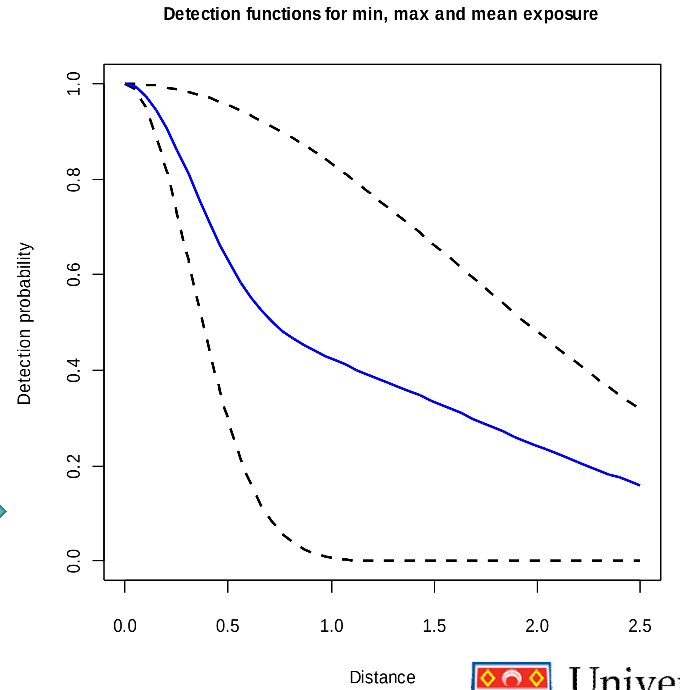


Scenario 1: animals have a gamma distribution of detection functions between min and max shown.

Mean estimate from simulation: 984 animals (SE 2.3). Bias -1.6%

Scenario 2: half of animals have max detection function, half have minimum.

Mean estimate from simulation: 976 animals (SE 2.7). Bias -2.4%



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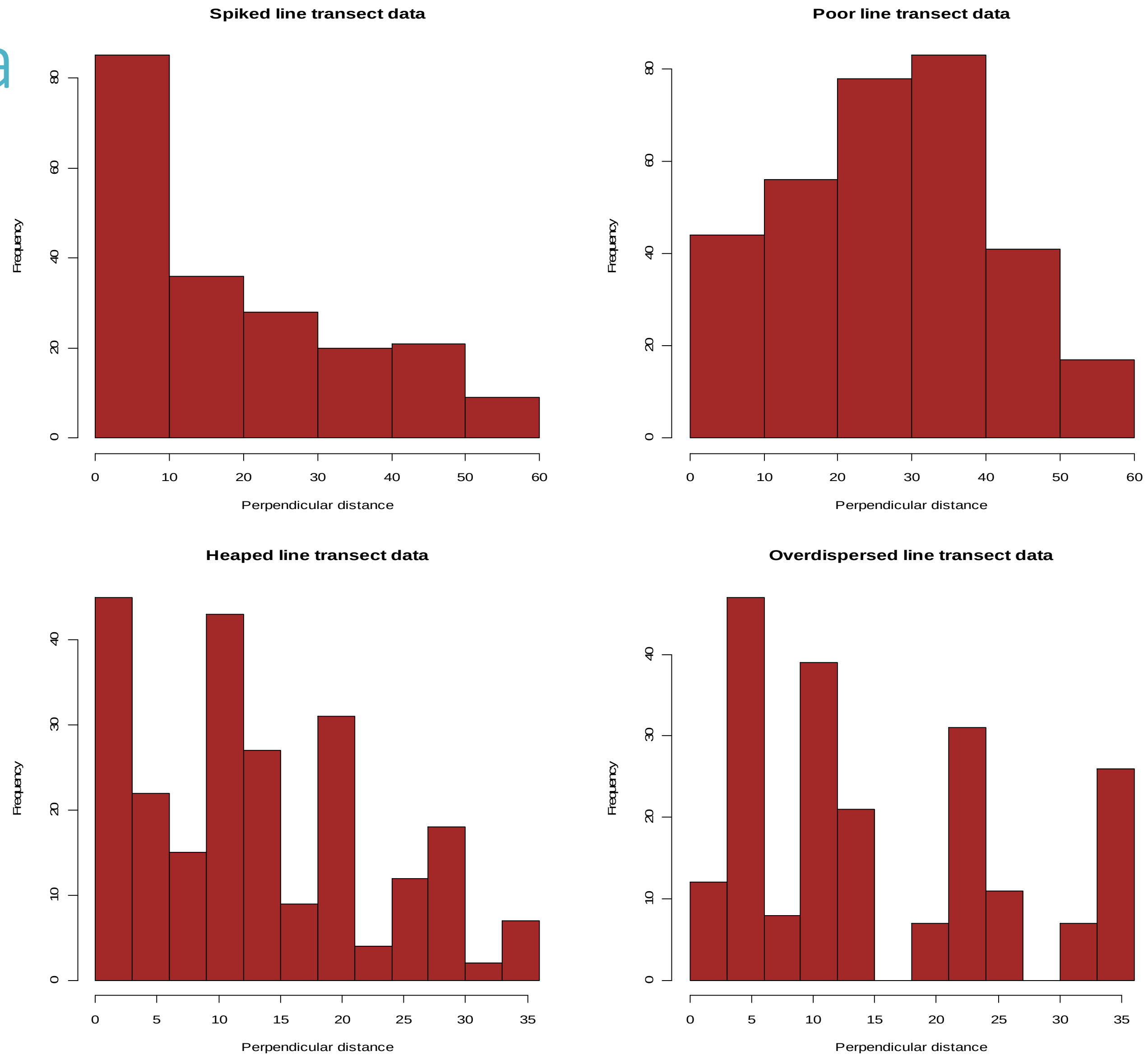
Non-ideal data



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