

# Multiple covariate distance sampling (MCDS)

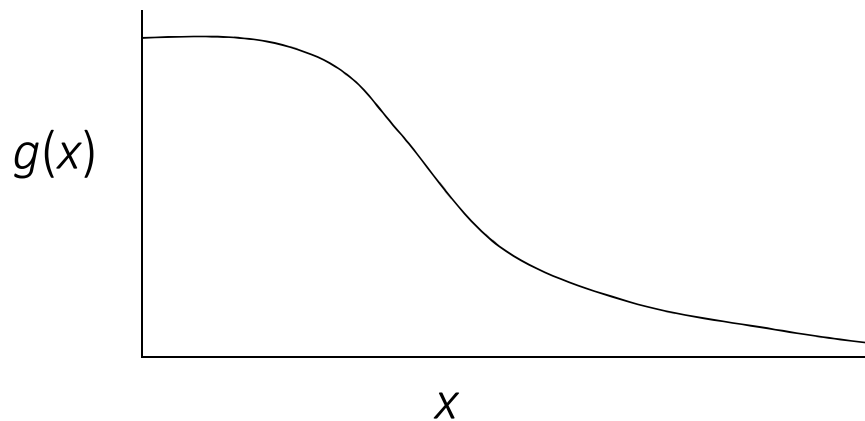
- Aim: Model the effect of additional covariates on detection probability, in addition to distance, while assuming probability of detection at zero distance is 1
- References:
  - Marques (F) and Buckland (2004) Covariate models for the detection function. Chapter 3 in Buckland *et al.* (eds). Advanced Distance Sampling.
  - Marques (T) *et al.* (2007) Improving estimates of bird density using multiple covariate distance sampling. *The Auk* 127: 1229-1243.
  - Section 5.3 of Buckland *et al.* (2015) Distance Sampling: Methods and Applications

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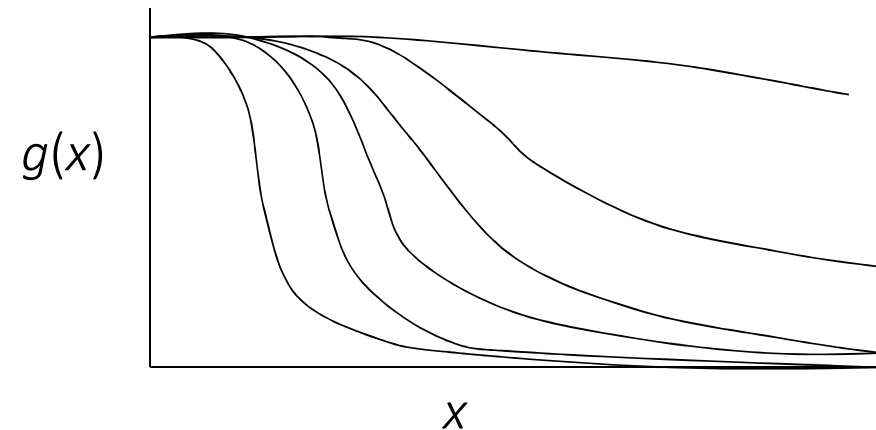
- Why additional covariates?
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# Why additional covariates?

In conventional distance sampling (CDS) analysis all factors affecting detectability, except distance, are ignored



In reality, many factors may affect detectability



Sources of heterogeneity:

Object : species, sex, cluster size

Effort: observer, habitat, weather

# Examples of heterogeneity 1

Effect of time of day on Rufous Fantail birds in Micronesia (point transects). Ramsey et. al. 1987. Biometrics 43:1-11

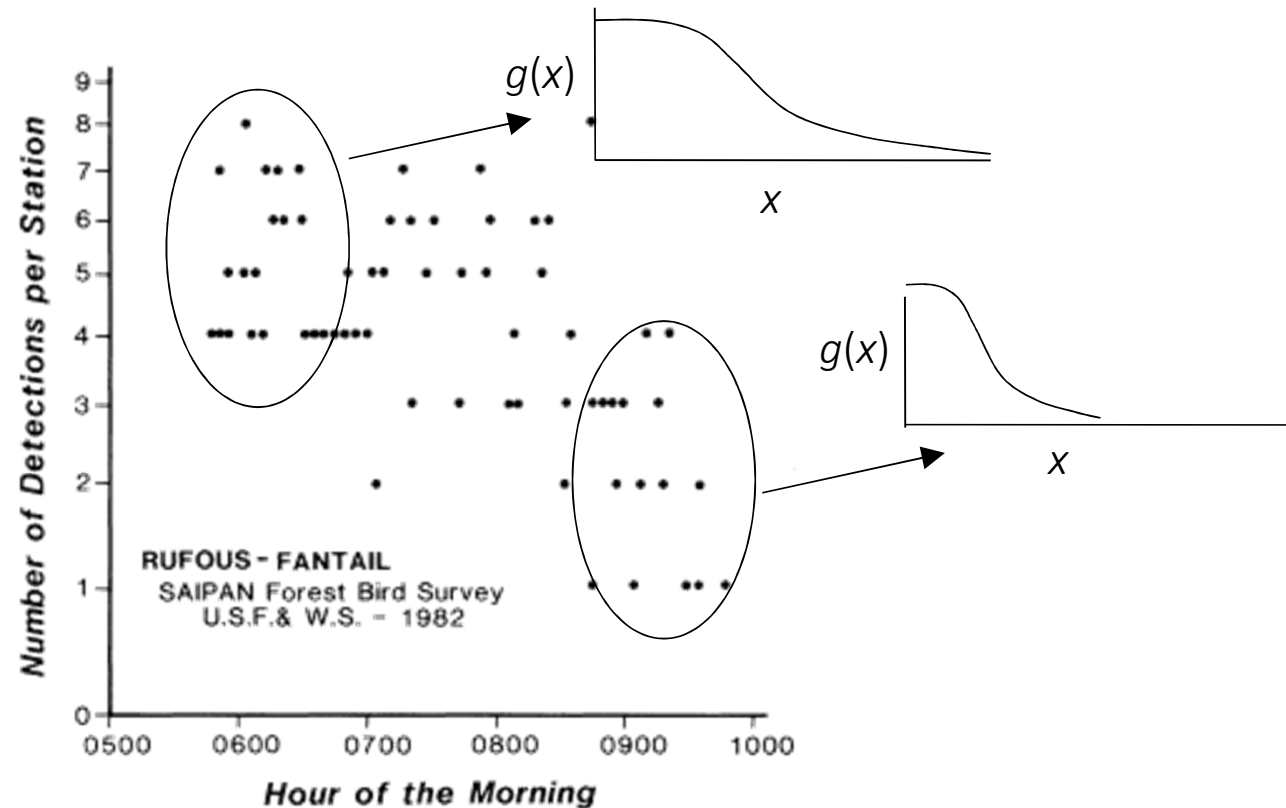


Figure 1. Station counts of Rufous Fantails on Saipan appear higher in the early morning hours than in the late morning ( $n = 64$ ,  $r = -.60$ ).

## Examples of heterogeneity 2

Effect of sea state (and other covariates) on sea turtles in the Eastern Tropical Pacific (shipboard line transects). Beavers and Ramsey, 1998, *J. Wildl. Manage.* 63: 948-957

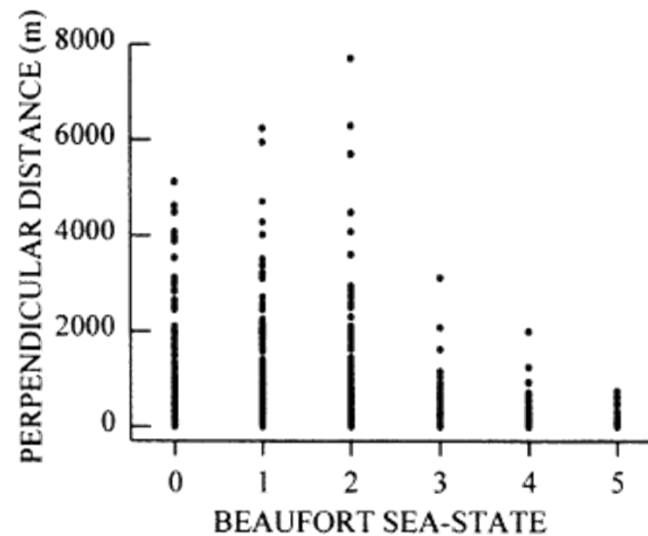
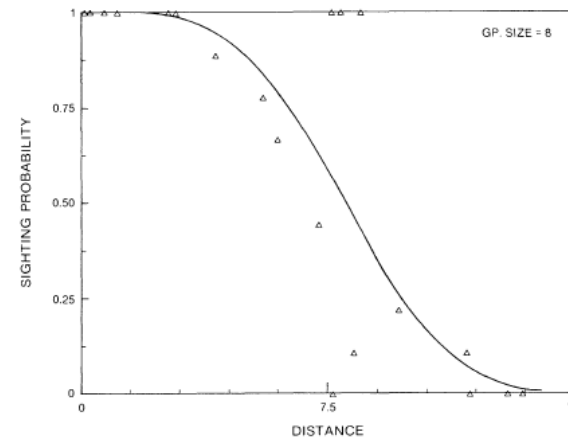
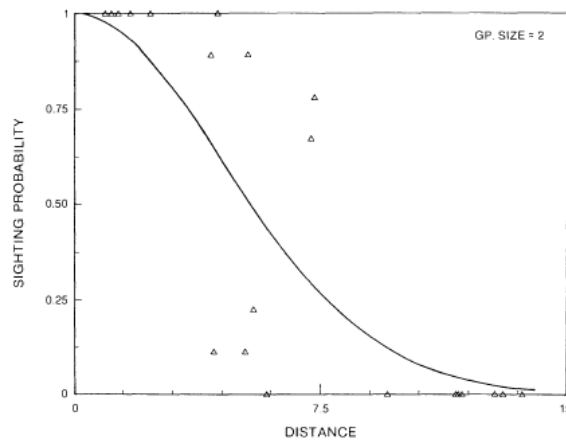
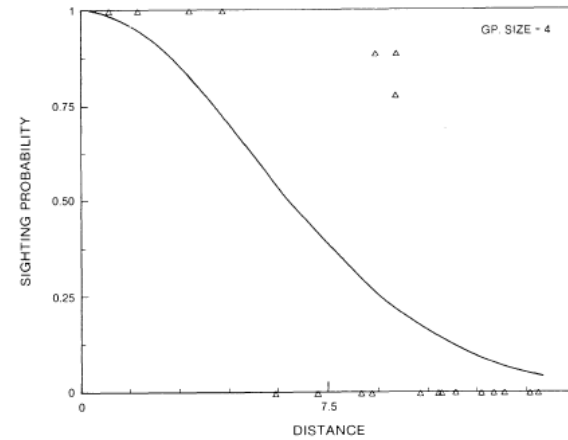
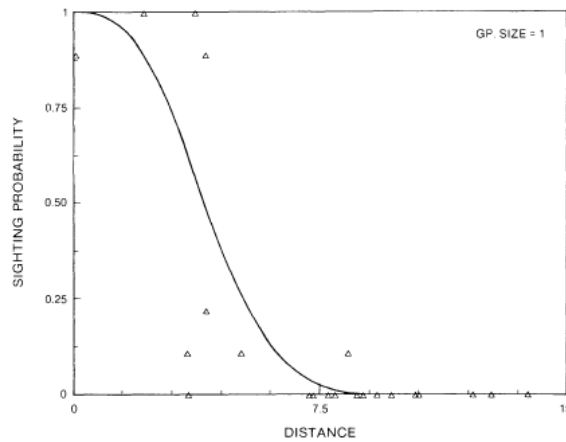


Fig. 2. Covariates of air temperature, sea surface temperature, and Beaufort sea-state plotted against unadjusted, ungrouped perpendicular sighting distances (m) of sea turtles in the eastern tropical Pacific, 1989–90.

# Examples of heterogeneity 3

Effect of cluster size on beer can detectability. Otto and Pollock, 1990, Biometrics 46: 239-245



# Why worry about heterogeneity?

In CDS, we use models that are pooling robust, so why worry about heterogeneity?

- Pooling robustness works for all but extreme levels of heterogeneity
- Potential bias if density is estimated at a 'lower level' than detection function (e.g. density by geographic region, detection function global)
- Could potentially increase precision of detection function estimate
- Interest in sources of heterogeneity in their own right (e.g. group size)

# Dealing with heterogeneity

## Stratification

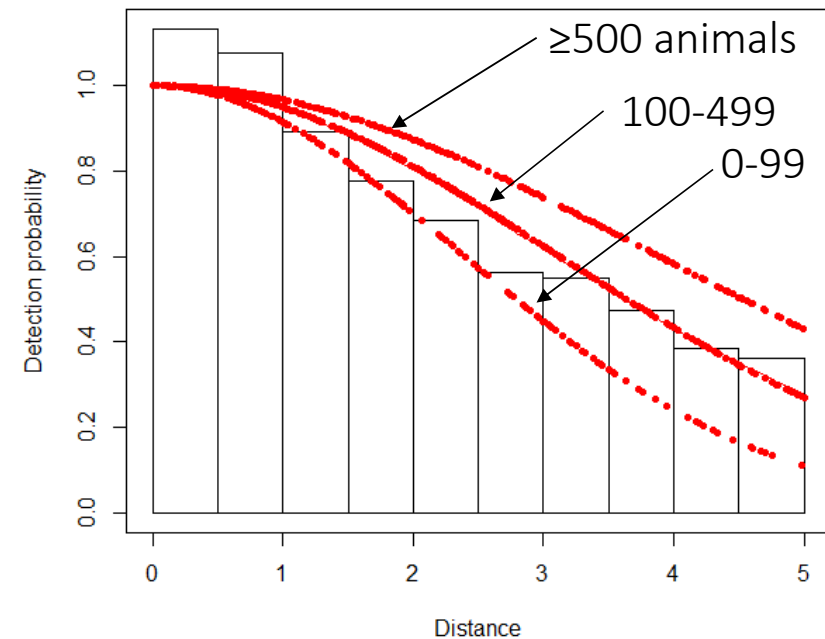
Requires estimating separate detection function parameters for each stratum,

- often not possible due to lack of data

## Model as covariates in detection function

Allows a more parsimonious approach:

- can model effect of numerical covariates
- can 'share information' about detection function *shape* between covariate levels





# Multiple covariate models Recap of CDS models

$g(x) = \text{Pr}[\text{animal at distance } x \text{ is detected}]$

$$= k(x) \left[ 1 + \sum_{j=1}^m a_j p_j(x_s) \right] / c$$

Key function

$j^{\text{th}}$  series adjustment term

Scaling constant to ensure  $g(0) = 1$

# CDS models continued

Key functions

Shape parameter

Hazard rate  $k(x) = 1 - \exp\left[-\left(\frac{x}{\sigma}\right)^{-b}\right]$

Half-normal  $k(x) = \exp\left(\frac{-x^2}{2\sigma^2}\right)$

Uniform  $k(x) = 1$

Scale parameter

Series adjustments

Cosine  $\cos(j\pi x_s)$

Polynomial  $x_s^j$

Hermite poly.  $H_j(x_s)$

$x_s$  are scaled distances

# Modelling with covariates


$g(x, z) = \text{Pr}[\text{animal at distance } x \text{ and covariates } z \text{ is detected}]$

Assume the covariates affect the *scale* of the key function, not its *shape*. So choose key functions with a scale parameter

Let 
$$\sigma(z) = \exp\left(\beta_0 + \sum_{j=1}^J \beta_j z_j\right)$$

e.g. Hazard rate 
$$k(x, z) = 1 - \exp\left[-\left(\frac{x}{\sigma(z)}\right)^{-b}\right]$$

Half normal 
$$k(x, z) = \exp\left(\frac{-x^2}{2\sigma(z)^2}\right)$$

  $k$  is used here to denote the “key” function

# Modelling with covariates

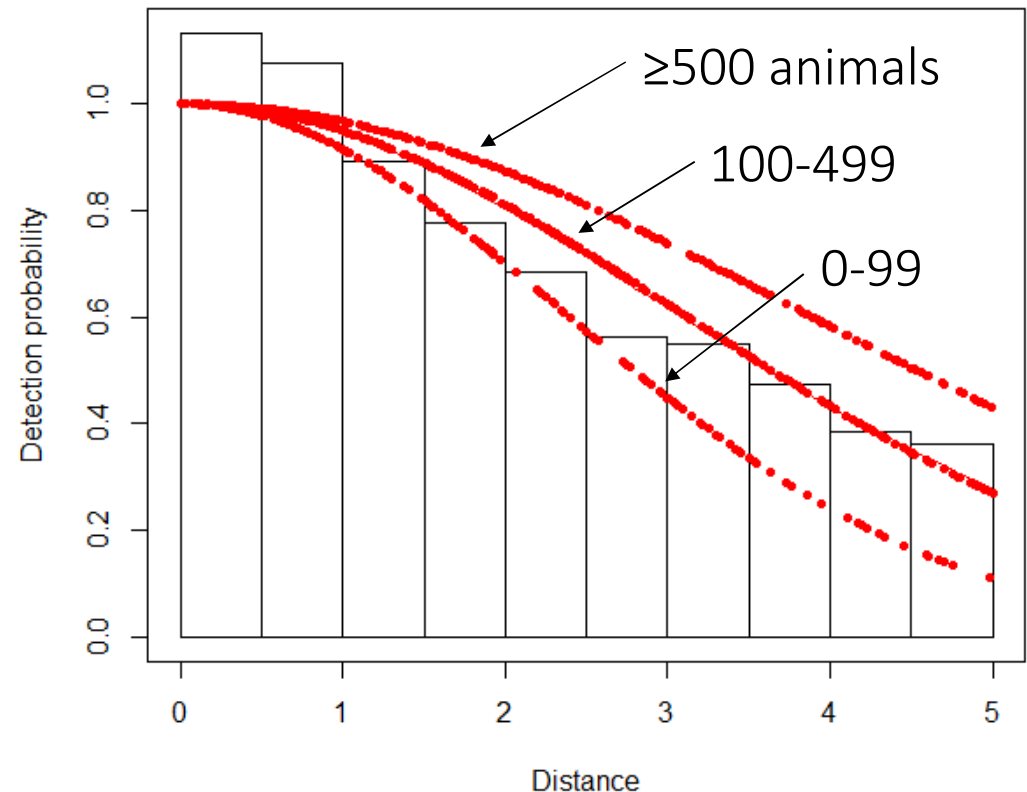
Example: Dolphin tuna vessel data

Model: half-normal, with no adjustments

Covariate: cluster size as factor (3 levels)  
with dummy variables,  $s_{d1}$  and  $s_{d2}$

$$g(x, s) = \exp\left(\frac{-x^2}{2\sigma(s)^2}\right)$$

$$\sigma(s) = \exp(\beta_0 + \beta_1 s_{d1} + \beta_2 s_{d2})$$



# Estimating abundance without covariates using Horvitz-Thompson estimator

$$\hat{N} = \sum_{i=1}^n \frac{1}{Pr[\textit{animal included}]} = \sum_{i=1}^n \frac{1}{\left[ \frac{2wL\hat{P}_a}{A} \right]} = \frac{nA}{2wL\hat{P}_a}$$

Recall that  $f(x)$  = pdf of observed  $x$ 's =  $\frac{g(x)}{\int g(x)dx} = \frac{g(x)}{\mu} = \frac{g(x)}{wP_a}$

Remember:  
 $x$ 's are the distances  
and  $P_a = \mu/w$

Because  $g(0)=1$  by assumption, then  $f(0) = g(0)/\mu = 1/\mu = 1/wP_a$

So 
$$\hat{N} = \frac{nA}{2wL\hat{P}_a} = \frac{nA}{2L} \cdot \hat{f}(0)$$

# Estimating abundance with covariates

$$\hat{N} = \sum_{i=1}^n \frac{1}{\Pr[\text{animal included}]} = \sum_{i=1}^n \frac{1}{\left[ \frac{2wL\hat{P}_a(z_i)}{A} \right]} = \frac{A}{2wL} \sum_{i=1}^n \frac{1}{\hat{P}_a(z_i)}$$

$$\text{Now } f(x|\mathbf{z}) = \frac{g(x,\mathbf{z})}{\int g(x,\mathbf{z})dx} = \frac{g(x,\mathbf{z})}{\mu(\mathbf{z})} = \frac{g(x,\mathbf{z})}{wP_a(\mathbf{z})}$$

Because  $g(0,\mathbf{z})=1$  by assumption, then  $f(0|\mathbf{z}) = \frac{g(0,\mathbf{z})}{\mu(\mathbf{z})} = \frac{1}{\mu(\mathbf{z})} = \frac{1}{wP_a(\mathbf{z})}$

So

$$\hat{N} = \frac{A}{2wL} \sum_{i=1}^n \frac{1}{\hat{P}_a(0|\mathbf{z}_i)} = \frac{A}{2L} \sum_{i=1}^n \hat{f}(0|\mathbf{z}_i)$$

Note similarity to CDS estimator

# MCDS in Distance

In `ds` command, specify covariates in `formula` argument

```
ds(data, key, formula)
```

E.g. `ds(data=Dolphin, key="hn", formula=~size.class)`

Covariate type:

- Factor covariates classify the data into distinct classes or levels. Can be numerical or text. One parameter per factor level.
- Non-factor (i.e., continuous) covariates must be numerical (integer or decimal). One parameter per covariate + 1 for the intercept.

# Complications 1. Clustered populations

When cluster size is a covariate:

- Distance recognizes cluster size because column is called `size` (i.e. reserved word)

E.g. `ds(data=Dolphin, key="hn", formula=~size)`

$$\hat{N}_{group} = \sum_{i=1}^n \frac{1}{Pr[group\ i\ included]} \quad \hat{N} = \sum_{i=1}^n \frac{size\ of\ group\ i}{Pr[group\ i\ included]}$$

Estimate of group size is given by  $\hat{E}[s] = \frac{\hat{N}}{\hat{N}_{group}}$



# MCDS analysis guidelines

Choose covariates that are:

- independent of distance
- not strongly correlated with each other

Specifying the model:

- factor covariates generally harder to fit
- check convergence and monotonicity
- add only one covariate at a time
- where necessary, use starting values and bounds for parameters
- consider reducing the truncation distance,  $w$ , if more than 5% of the  $P_a(z_i)$  are  $<0.2$ , or if any are less than 0.1